CHAPTER 1

Functions from a Calculus Perspective

Then

In **Algebra 2**, you analyzed functions from a graphical perspective.

Now

In **Chapter 1**, you will:

- Explore symmetries of graphs.
- Determine continuity and average rates of change of functions.
- Use limits to describe end behavior.
- Find inverse functions algebraically and graphically.

Why? ▲

BUSINESS Functions are often used throughout the business world. Some of the uses of functions are to analyze costs, predict sales, calculate profit, forecast future costs and revenue, estimate depreciation, and determine the proper labor force.

PREREAD Create a list of two or three things that you already know about functions. Then make a prediction of what you will learn in Chapter 1.

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Get Ready for the Chapter

Diagnose Readiness You have two options for checking Prerequisite Skills.

1 Textbook Option Take the Quick Check below.

QuickCheck

Graph each inequality on a number line. (Prerequisite Skill)

1. $x > -3$ or $x < -6$

2. $-4 < x \le -2$

3. $-1 \le x \le 5$

4. $x < -4$ or $x > 1$

5. $5 \le x - 4 \le 12$

6. $7 > -x > 2$

Solve each equation for y. (Prerequisite Skill)

7. $y - 3x = 2$

8. $y + 4x = -5$

9. $2x - y^2 = 7$

10. $y^2 + 5 = -3x$

11. $9 + y^3 = -x$

12. $y^3 - 9 = 11x$

13. **TEMPERATURE** The formula $C = \frac{5}{9}(F - 32)$, where C represents a temperature in degrees Celsius and F in degrees Fahrenheit, can be used to convert between the two measures. If the temperature on a thermometer reads 23°C, what is the temperature in degrees Fahrenheit rounded to the nearest tenth? (Prerequisite Skill)

Evaluate each expression given the value of the variable. (Prerequisite Skill)

14. $3y^2 - 4$, $y = 2$

15. $2b^3 + 7$, $b = -3$

16. $x^2 + 2x - 3$, $x = -4a$

17. $5z - 2z^2 + 1$, $z = 5x$

18. $-4c^2 + 7$, $c = 7a^2$

19. $2 + 3p^2$, $p = -5 + 2n$

20. **PROJECTILES** Two students are launching a model rocket for science class. The height of the rocket can be modeled by the function $h(t) = -16t^2 + 200t + 26$, where t is time in seconds and h is the height in feet. Find the height of the rocket after 7 seconds. (Prerequisite Skill)

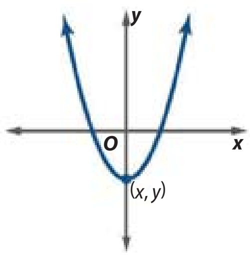
2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

NewVocabulary

English		Español
interval notation	p. 5	notación del intervalo
function	p. 5	función
function notation	p. 7	notación de la función
implied domain	p. 7	dominio implicado
zeros	p. 15	ceros
roots	p. 15	raíz
even function	p. 18	función uniforme
odd function	p. 18	función impar
limit	p. 24	límite
end behavior	p. 28	comportamiento final
increasing	p. 34	aumento
decreasing	p. 34	el disminuir
constant	p. 34	constante
maximum	p. 36	máximo
minimum	p. 36	mínimo
extrema	p. 36	extrema
secant line	p. 38	línea secante
parent function	p. 45	función del padre
transformation	p. 46	transformación
reflection	p. 48	reflexión
dilation	p. 49	dilatación
composition	p. 58	composición

ReviewVocabulary

parabola p. P9 parábola the graph of a quadratic function



slope • Prerequisite Skill • línea pendiente the ratio of the change in y-coordinates to the change in x-coordinates

LESSON 1-1

Functions

Then

- You used set notation to denote elements, subsets, and complements. (Lesson 0-1)

Now

1. Describe subsets of real numbers.
2. Identify and evaluate functions and state their domains.

Why?

- Many events that occur in everyday life involve two related quantities. For example, to operate a vending machine, you insert money and make a selection. The machine gives you the selected item and any change due. Once your selection is made, the amount of change you receive *depends* on the amount of money you put into the machine.

NewVocabulary

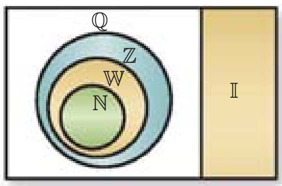
set-builder notation
interval notation
function
function notation
independent variable
dependent variable
implied domain
piecewise-defined function
relevant domain

1 Describe Subsets of Real Numbers

Real numbers are used to describe quantities such as money and distance. The set of real numbers $\mathbb{R}$ includes the following subsets of numbers.

KeyConcept Real Numbers

Real Numbers ($\mathbb{R}$)



Letter	Set	Examples
$\mathbb{Q}$	rationals	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.666\ldots$
$\mathbb{I}$	irrationals	$\sqrt{3} = 1.73205\ldots$
$\mathbb{Z}$	integers	$-5, 17, -23, 8$
$\mathbb{W}$	wholes	$0, 1, 2, 3\ldots$
$\mathbb{N}$	naturals	$1, 2, 3, 4\ldots$

These and other sets of real numbers can be described using set-builder notation. **Set-builder notation** uses the properties of the numbers in the set to define the set.

$$\{x \mid -3 \leq x \leq 16,\ x \in \mathbb{Z}\}$$

The set of numbers x such that… x has the given properties… and x is an element of the given set of numbers.

Example 1 Use Set Builder Notation

Describe the set of numbers using set-builder notation.

a. $\{8, 9, 10, 11, \ldots\}$

The set includes all whole numbers greater than or equal to 8.

$\{x \mid x \geq 8, x \in \mathbb{W}\}$ Read as *the set of all x such that x is greater than or equal to 8 and x is an element of the set of whole numbers.*

b. $x < 7$

Unless otherwise stated, you should assume that a given set consists of real numbers. Therefore, the set includes all real numbers less than 7. $\{x \mid x < 7, x \in \mathbb{R}\}$

c. all multiples of three

The set includes all integers that are multiples of three. $\{x \mid x = 3n, n \in \mathbb{Z}\}$

GuidedPractice

1A. $\{1, 2, 3, 4, 5, \ldots\}$ **1B.** $x \leq -3$ **1C.** all multiples of π

Flirt/SuperStock

Study Tip

Look Back You can review set notation, including unions and intersections of sets, in Lesson 0-1.

Interval notation uses inequalities to describe subsets of real numbers. The symbols [or] are used to indicate that an endpoint is included in the interval, while the symbols (or) are used to indicate that an endpoint is not included in the interval. The symbols ∞, positive infinity, and $-\infty$, negative infinity, are used to describe the unboundedness of an interval. An interval is *unbounded* if it goes on indefinitely.

Bounded Intervals		Unbounded Intervals	
Inequality	Interval Notation	Inequality	Interval Notation
$a \le x \le b$	$[a, b]$	$x \ge a$	$[a, \infty)$
$a < x < b$	(a, b)	$x \le a$	$(-\infty, a]$
$a \le x < b$	$[a, b)$	$x > a$	(a, ∞)
$a < x \le b$	$(a, b]$	$x < a$	$(-\infty, a)$
		$-\infty < x < \infty$	$(-\infty, \infty)$

Example 2 Use Interval Notation

Write each set of numbers using interval notation.

a. $-8 < x \le 16$ $\quad (-8, 16]$

b. $x < 11$ $\quad (-\infty, 11)$

c. $x \le -16$ **or** $x > 5$ $\quad (-\infty, -16] \cup (5, \infty)$ $\quad$ $\cup$ read as *union*

Guided Practice

2A. $-4 \le y < -1$ **2B.** $a \ge -3$ **2C.** $x > 9$ or $x < -2$

2 Identify Functions

Recall that a *relation* is a rule that relates two quantities. Such a rule pairs the elements in a set A with elements in a set B. The set A of all inputs is the *domain* of the relation, and set B contains all outputs or the *range*.

Relations are commonly represented in four ways.

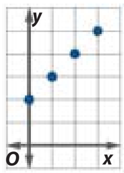
1. **Verbally** A sentence describes how the inputs and outputs are related.

   The output value is 2 more than the input value.

2. **Numerically** A table of values or a set of ordered pairs relates each input (x-value) with an output value (y-value).

   $\{(0, 2), (1, 3), (2, 4), (3, 5)\}$

3. **Graphically** Points on a graph in the coordinate plane represent the ordered pairs.



4. **Algebraically** An equation relates the x- and y-coordinates of each ordered pair.

   $y = x + 2$

A **function** is a special type of relation.

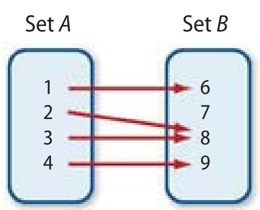
Key Concept Function

Words A function f from set A to set B is a relation that assigns to each element x in set A *exactly one* element y in set B.

Symbols The relation from set A to set B is a function.

Set A is the domain. $\quad D = \{1, 2, 3, 4\}$

Set B contains the range. $\quad R = \{6, 8, 9\}$



Study Tip

Domain and Range In this text, the notation for domain and range will be D = and R =, respectively.

StudyTip

Tabular Method When a relation fails the vertical line test, an x-value has more than one corresponding y-value, as shown below.

x	y
−2	−4
3	−1
3	4
5	6
7	9

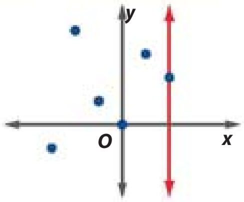
An alternate definition of a function is a set of ordered pairs in which no two different pairs have the same x-value. Interpreted graphically, this means that no two points on the graph of a function in the coordinate plane can lie on the same vertical line.

KeyConcept Vertical Line Test

Words

A set of points in the coordinate plane is the graph of a function if each possible vertical line intersects the graph in at most one point.

Model



Example 3 Identify Relations that are Functions

Determine whether each relation represents y as a function of x.

a. The input value x is a student's ID number, and the output value y is that student's score on a physics exam.

Each value of x cannot be assigned to more than one y-value. A student cannot receive two different scores on an exam. Therefore, the sentence describes y as a function of x.

StudyTip

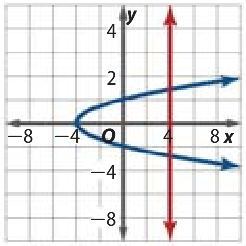
Functions with Repeated y-Values While a function *cannot* have more than one y-value paired with each x-value, a function *can* have one y-value paired with more than one x-value, as shown in Example 3b.

b.

x	y
−8	−5
−5	−4
0	−3
3	−2
6	−3

Each x-value is assigned to exactly one y-value. Therefore, the table represents y as a function of x.

c.



A vertical line at $x = 4$ intersects the graph at more than one point. Therefore, the graph does not represent y as a function of x.

d. $y^2 - 2x = 5$

To determine whether this equation represents y as a function of x, solve the equation for y.

$y^2 - 2x = 5$ Original equation

$y^2 = 5 + 2x$ Add $2x$ to each side.

$y = \pm\sqrt{5 + 2x}$ Take the square root of each side.

This equation does not represent y as a function of x because there will be two corresponding y-values, one positive and one negative, for any x-value greater than −2.5.

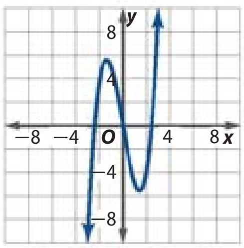
GuidedPractice

3A. The input value x is the area code, and the output value y is a phone number in that area code.

3B.

x	y
−6	−7
2	3
5	8
5	9
9	22

3C.



3D. $3y + 6x = 18$

In **function notation**, the symbol $f(x)$ is read f of x and interpreted as *the value of the function f at x*. Because $f(x)$ corresponds to the y-value of f for a given x-value, you can write $y = f(x)$.

Equation	Related Function
$y = -6x$	$f(x) = -6x$

Because it can represent any value in the function's domain, x is called the **independent variable**. A value in the range of f is represented by the **dependent variable**, y.

Example 4 Find Function Values

If $g(x) = x^2 + 8x - 24$, find each function value.

a. $g(6)$

To find $g(6)$, replace x with 6 in $g(x) = x^2 + 8x - 24$.

$$
\begin{aligned}
g(x) &= x^2 + 8x - 24 && \text{Original function}\\
g(6) &= (6)^2 + 8(6) - 24 && \text{Substitute 6 for } x.\\
&= 36 + 48 - 24 && \text{Simplify.}\\
&= 60 && \text{Simplify.}
\end{aligned}
$$

b. $g(-4x)$

$$
\begin{aligned}
g(x) &= x^2 + 8x - 24 && \text{Original function}\\
g(-4x) &= (-4x)^2 + 8(-4x) - 24 && \text{Substitute } -4x \text{ for } x.\\
&= 16x^2 - 32x - 24 && \text{Simplify.}
\end{aligned}
$$

c. $g(5c + 4)$

$$
\begin{aligned}
g(x) &= x^2 + 8x - 24 && \text{Original function}\\
g(5c + 4) &= (5c + 4)^2 + 8(5c + 4) - 24 && \text{Substitute } 5c + 4 \text{ for } x.\\
&= 25c^2 + 40c + 16 + 40c + 32 - 24 && \text{Expand } (5c + 4)^2 \text{ and } 8(5c + 4).\\
&= 25c^2 + 80c + 24 && \text{Simplify.}
\end{aligned}
$$

GuidedPractice

If $f(x) = \dfrac{2x + 3}{x^2 - 2x + 1}$, find each function value.

4A. $f(12)$ **4B.** $f(6x)$ **4C.** $f(-3a + 8)$

Math HistoryLink

Leonhard Euler
(1707–1783)
A Swiss mathematician, Euler was a prolific mathematical writer, publishing over 800 papers in his lifetime. He also introduced much of our modern mathematical notation, including the use of $f(x)$ for the function f.

When you are given a function with an unspecified domain, the **implied domain** is the set of all real numbers for which the expression used to define the function is real. In general, you must exclude values from the domain of a function that would result in division by zero or taking the even root of a negative number.

Example 5 Find Domains Algebraically

State the domain of each function.

a. $f(x) = \dfrac{2 + x}{x^2 - 7x}$

When the denominator of $\dfrac{2 + x}{x^2 - 7x}$ is zero, the expression is undefined. Solving $x^2 - 7x = 0$, the excluded values for the domain of this function are $x = 0$ and $x = 7$. The domain of this function is all real numbers except $x = 0$ and $x = 7$, or $\{x \mid x \neq 0, x \neq 7, x \in \mathbb{R}\}$.

b. $g(t) = \sqrt{t - 5}$

Because the square root of a negative number cannot be real, $t - 5 \geq 0$. Therefore, the domain of $g(t)$ is all real numbers t such that $t \geq 5$ or $[5, \infty)$.

StudyTip

Naming Functions You can use other letters to name a function and its independent variable. For example, $f(x) = \sqrt{x - 5}$ and $g(t) = \sqrt{t - 5}$ name the same function.

c. $h(x) = \dfrac{1}{\sqrt{x^2 - 9}}$

This function is defined only when $x^2 - 9 > 0$. Therefore, the domain of $h(x)$ is $(-\infty, -3) \cup (3, \infty)$.

GuidedPractice

State the domain of each function.

5A. $f(x) = \dfrac{5x - 2}{x^2 + 7x + 12}$ **5B.** $h(a) = \sqrt{a^2 - 4}$ **5C.** $g(x) = \dfrac{8x}{\sqrt{2x + 6}}$

A function that is defined using two or more equations for different intervals of the domain is called a **piecewise-defined function**.

Real-World Link

Robert Pershing Wadlow of Alton, Illinois, was the tallest man recorded in medical history at 8 feet 11.1 inches. Wadlow weighed almost 440 pounds.

Source: *Guinness Book of World Records*

Real-World Example 6 Evaluate a Piecewise-Defined Function

HEIGHT The average maximum height of children in inches as a function of their parents' maximum heights in inches can be modeled by the following piecewise function. Find the average maximum heights of children whose parents have the given maximum heights. Use $h(x)$, where x is the independent variable representing the parents' height and $h(x)$ is the dependent variable representing the child's height.

$$h(x) = \begin{cases} 1.6x - 41.6 & \text{if} \quad 63 < x < 66 \\ 3x - 132 & \text{if} \quad 66 \le x \le 68 \\ 2x - 66 & \text{if} \quad x > 68 \end{cases}$$

a. $h(67)$

Because 67 is between 66 and 68, use $h(x) = 3x - 132$ to find $h(67)$.

$h(67) = 3x - 132$	Function for $66 \le x \le 68$
$= 3(67) - 132$	Substitute 67 for x.
$= 201 - 132$ or 69	Simplify.

According to this model, children whose parents have a maximum height of 67 inches will attain an average maximum height of 69 inches.

b. $h(72)$

Because 72 is greater than 68, use $h(x) = 2x - 66$.

$h(72) = 2x - 66$	Function for $x > 68$
$= 2(72) - 66$	Substitute 72 for x.
$= 144 - 66$ or 78	Simplify.

According to this model, children whose parents have a maximum height of 72 inches will attain an average maximum height of 78 inches.

StudyTip

Relevant Domain A **relevant domain** is the part of a domain that is relevant to a model. Consider a function in which the output is a function of length. It is unreasonable to have a negative length, so the relevant domain is the set of numbers greater than or equal to 0.

GuidedPractice

6. SPEED The speed v of a vehicle in miles per hour can be represented by the following piecewise function when t is the time in seconds. Find the speed of the vehicle at each indicated time.

$$v(t) = \begin{cases} 4t & \text{if} \quad 0 \le t \le 15 \\ 60 & \text{if} \quad 15 < t < 240 \\ -6t + 1500 & \text{if} \quad 240 \le t \le 250 \end{cases}$$

A. $v(5)$ **B.** $v(15)$ **C.** $v(245)$

Exercises

= Step-by-Step Solutions begin on page R29.

Write each set of numbers in set-builder and interval notation, if possible. (Examples 1 and 2)

1. $x > 50$
2. $x < -13$
3. $x \leq -4$
4. $\{-4, -3, -2, -1, \ldots\}$
5. $8 < x < 99$
6. $-31 < x \leq 64$
7. $x < -19$ or $x > 21$
8. $x < 0$ or $x \geq 100$
9. $\{-0.25, 0, 0.25, 0.50, \ldots\}$
10. $x \leq 61$ or $x \geq 67$
11. $x \leq -45$ or $x > 86$
12. all multiples of 8
13. all multiples of 5
14. $x \geq 32$

Determine whether each relation represents y as a function of x. (Example 3)

15. The input value x is a bank account number and the output value y is the account balance.

16. The input value x is the year and the output value y is the day of the week.

17.

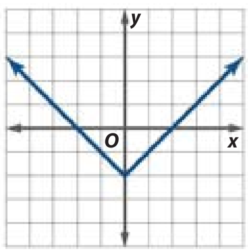
x	y
−50	2.11
−40	2.14
−30	2.16
−20	2.17
−10	2.17
0	2.18

18.

x	y
0.01	423
0.04	449
0.04	451
0.07	466
0.08	478
0.09	482

19. $\frac{1}{x} = y$
20. $x^2 = y + 2$
21. $3y + 4x = 11$
22. $4y^2 + 18 = 96x$
23. $\sqrt{48y} = x$
24. $\frac{x}{y} = y - 6$

25.



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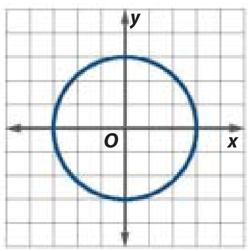

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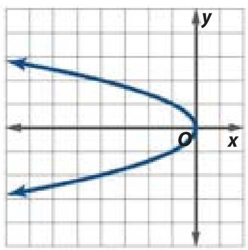

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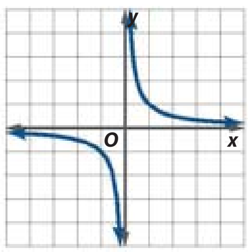
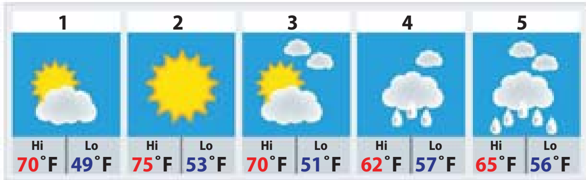


29. **METEOROLOGY** The five-day forecast for a city is shown. (Example 3)



a. Represent the relation between the day of the week and the estimated high temperature as a set of ordered pairs.

b. Is the estimated high temperature a function of the day of the week? the low temperature? Explain your reasoning.

Find each function value. (Example 4)

30. $g(x) = 2x^2 + 18x - 14$
  a. $g(9)$
  b. $g(3x)$
  c. $g(1 + 5m)$

31. $h(y) = -3y^3 - 6y + 9$
  a. $h(4)$
  b. $h(-2y)$
  c. $h(5b + 3)$

32. $f(t) = \frac{4t + 11}{3t^2 + 5t + 1}$
  a. $f(-6)$
  b. $f(4t)$
  c. $f(3 - 2a)$

33. $g(x) = \frac{3x^3}{x^2 + x - 4}$
  a. $g(-2)$
  b. $g(5x)$
  c. $g(8 - 4b)$

34. $h(x) = 16 - \frac{12}{2x + 3}$
  a. $h(-3)$
  b. $h(6x)$
  c. $h(10 - 2c)$

35. $f(x) = -7 + \frac{6x + 1}{x}$
  a. $f(5)$
  b. $f(-8x)$
  c. $f(6y + 4)$

36. $g(m) = 3 + \sqrt{m^2 - 4}$
  a. $g(-2)$
  b. $g(3m)$
  c. $g(4m - 2)$

37. $t(x) = 5\sqrt{6x^2}$
  a. $t(-4)$
  b. $t(2x)$
  c. $t(7 + n)$

38. **DIGITAL AUDIO PLAYERS** The sales of digital audio players in millions of dollars for a five-year period can be modeled using $f(t) = 24t^2 - 93t + 78$, where t is the year. The actual sales data are shown in the table. (Example 4)

Year	Sales ($)
1	1 million
2	3 million
3	14 million
4	74 million
5	219 million

a. Find $f(1)$ and $f(5)$.

b. Do you think that the model is more accurate for the earlier years or the later years? Explain your reasoning.

State the domain of each function. (Example 5)

39. $f(x) = \dfrac{8x + 12}{x^2 + 5x + 4}$

40. $g(x) = \dfrac{x + 1}{x^2 - 3x - 40}$

41. $g(a) = \sqrt{1 + a^2}$

42. $h(x) = \sqrt{6 - x^2}$

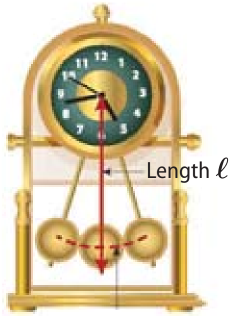
43. $f(a) = \dfrac{5a}{\sqrt{4a - 1}}$

44. $g(x) = \dfrac{3}{\sqrt{x^2 - 16}}$

45. $f(x) = \dfrac{2}{x} + \dfrac{4}{x + 1}$

46. $g(x) = \dfrac{6}{x + 3} + \dfrac{2}{x - 4}$

47. PHYSICS The period T of a pendulum is the time for one cycle and can be calculated using the formula $T = 2\pi\sqrt{\frac{\ell}{9.8}}$, where ℓ is the length of the pendulum and 9.8 is the acceleration due to gravity in meters per second squared. Is this formula a function of ℓ? If so, determine the domain. If not, explain why not. (Example 5)



Find $f(-5)$ and $f(12)$ for each piecewise function. (Example 6)

48. $f(x) = \begin{cases} -4x + 3 & \text{if } x < 3 \\ -x^3 & \text{if } 3 \le x \le 8 \\ 3x^2 + 1 & \text{if } x > 8 \end{cases}$

49. $f(x) = \begin{cases} -5x^2 & \text{if } x < -6 \\ x^2 + x + 1 & \text{if } -6 \le x \le 12 \\ 0.5x^3 - 4 & \text{if } x > 12 \end{cases}$

50. $f(x) = \begin{cases} 2x^2 + 6x + 4 & \text{if } x < -4 \\ 6 - x^2 & \text{if } -4 \le x < 12 \\ 14 & \text{if } x \ge 12 \end{cases}$

51. $f(x) = \begin{cases} -15 & \text{if } x < -5 \\ \sqrt{x + 6} & \text{if } -5 \le x \le 10 \\ \frac{2}{x} + 8 & \text{if } x > 10 \end{cases}$

52. INCOME TAX Federal income tax for a person filing single in the United States in a recent year can be modeled using the following function, where x represents income and $T(x)$ represents total tax. (Example 6)

$$T(x) = \begin{cases} 0.10x & \text{if } 0 \le x \le 7285 \\ 782.5 + 0.15x & \text{if } 7285 < x \le 31{,}850 \\ 4386.25 + 0.25x & \text{if } 31{,}850 < x \le 77{,}100 \end{cases}$$

a. Find $T(7000)$, $T(10{,}000)$, and $T(50{,}000)$.

b. If a person's annual income were \$7285, what would his or her income tax be?

53. PUBLIC TRANSPORTATION The nationwide use of public transportation can be modeled using the following function. The year 2012 is represented by $t = 0$, and $P(t)$ represents passenger trips in millions. (Example 6)

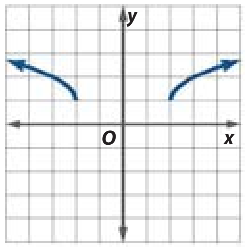
$$P(t) = \begin{cases} 0.35t + 7.6 & \text{if } 0 \le t \le 5 \\ 0.04t^2 - 0.6t + 11.6 & \text{if } 5 < t \le 10 \end{cases}$$

a. Approximately how many passenger trips were there in 2016? in 2020?

b. State the domain of the function.

Use the vertical line test to determine whether each graph represents a function. Write *yes* or *no*. Explain your reasoning.

54.



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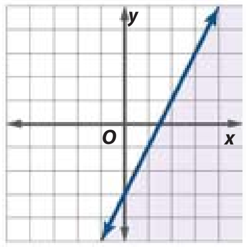

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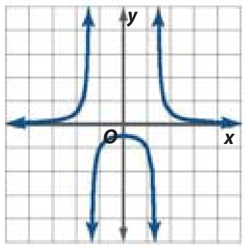

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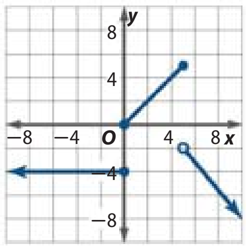


58. TRIATHLON In a triathlon, athletes swim 2.4 miles, then bike 112 miles, and finally run 26.2 miles. Jesse's average rates for each leg of a triathlon are shown in the table.

Leg	Rate
swim	4 mph
bike	20 mph
run	6 mph

a. Write a piecewise function to describe the distance D that Jesse has traveled in terms of time t. Round t to the nearest tenth, if necessary.

b. State the domain of the function.

59. ELECTIONS Describe the set of presidential election years beginning in 1792 in interval notation or in set-builder notation. Explain your reasoning.

60. CONCESSIONS The number of students working the concession stands at a football game can be represented by $f(x) = \frac{x}{50}$, where x is the number of tickets sold. Describe the relevant domain of the function.

61. **ATTENDANCE** The Chicago Cubs franchise has been in existence since 1876. The total season attendance for its home games can be modeled by $f(x) = 21{,}870x - 40{,}962{,}679$, where x represents the year. Describe the relevant domain of the function.

62. **ACCOUNTING** A business' assets, such as equipment, wear out or depreciate over time. One way to calculate depreciation is the straight-line method, using the value of the estimated life of the asset. Suppose $v(t) = 10{,}440 - 290t$ describes the value $v(t)$ of a copy machine after t months. Describe the relevant domain of the function.

Find $f(a)$, $f(a + h)$, and $\frac{f(a+h) - f(a)}{h}$ if $h \neq 0$.

63. $f(x) = -5$

64. $f(x) = \sqrt{x}$

65. $f(x) = \frac{1}{x+4}$

66. $f(x) = \frac{2}{5-x}$

67. $f(x) = x^2 - 6x + 8$

68. $f(x) = -\frac{1}{4}x + 6$

69. $f(x) = -x^5$

70. $f(x) = x^3 + 9$

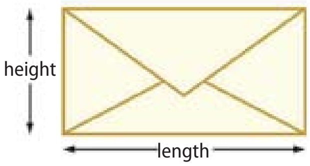
71. $f(x) = 7x - 3$

72. $f(x) = 5x^2$

73. $f(x) = x^3$

74. $f(x) = 11$

75. **MAIL** The U.S. Postal Service requires that envelopes have an aspect ratio (length divided by height) of 1.3 to 2.5, inclusive. The minimum allowable length is 5 inches and the maximum allowable length is $11\frac{1}{2}$ inches.

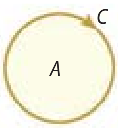


a. Write the area of the envelope A as a function of length ℓ if the aspect ratio is 1.8. State the domain of the function.

b. Write the area of the envelope A as a function of height h if the aspect ratio is 2.1. State the domain of the function.

c. Find the area of an envelope with the maximum height at the maximum aspect ratio.

76. **GEOMETRY** Consider the circle below with area A and circumference C.



a. Represent the area of the circle as a function of its circumference.

b. Find $A(0.5)$ and $A(4)$.

c. What do you notice about the area as the circumference increases?

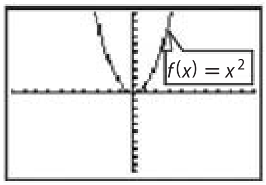
Determine whether each equation is a function of x. Explain.

77. $x = |y|$

78. $x = y^3$

79. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the range of a function.

a. **GRAPHICAL** Use a graphing calculator to graph $f(x) = x^n$ for whole-number values of n from 1 to 6, inclusive.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

b. **TABULAR** Predict the range of each function based on the graph, and tabulate each value of n and the corresponding range.

c. **VERBAL** Make a conjecture about the range of $f(x)$ when n is even.

d. **VERBAL** Make a conjecture about the range of $f(x)$ when n is odd.

H.O.T. Problems Use Higher-Order Thinking Skills

80. **ERROR ANALYSIS** Ana and Mason are evaluating $f(x) = \frac{2}{x^2 - 4}$. Ana thinks that the domain of the function is $(-\infty, -2) \cup (1, 1) \cup (2, \infty)$. Mason thinks that the domain is $\{x \mid x \neq -2, x \neq 2, x \in \mathbb{R}\}$. Is either of them correct? Explain.

81. **WRITING IN MATH** Write the domain of $f(x) = \frac{1}{(x+3)(x+1)(x-5)}$ in interval notation and in set-builder notation. Which notation do you prefer? Explain.

82. **CHALLENGE** $G(x)$ is a function for which $G(1) = 1$, $G(2) = 2$, $G(3) = 3$, and $G(x + 1) = \frac{G(x-2)\,G(x-1) + 1}{G(x)}$ for $x \geq 3$. Find $G(6)$.

REASONING **Determine whether each statement is *true* or *false* given a function from set X to set Y. If a statement is false, rewrite it to make a true statement.**

83. Every element in X must be matched with only one element in Y.

84. Every element in Y must be matched with an element in X.

85. Two or more elements in X may not be matched with the same element in Y.

86. Two or more elements in Y may not be matched with the same element in X.

WRITING IN MATH **Explain how you can identify a function described as each of the following.**

87. a verbal description of inputs and outputs

88. a set of ordered pairs

89. a table of values

90. a graph

91. an equation

Spiral Review

Find the standard deviation of each population of data. (Lesson 0-8)

92. {200, 476, 721, 579, 152, 158}

93. {5.7, 5.7, 5.6, 5.5, 5.3, 4.9, 4.4, 4.0, 4.0, 3.8}

94. {369, 398, 381, 392, 406, 413, 376, 454, 420, 385, 402, 446}

95. BASEBALL How many different 9-player teams can be made if there are 3 players who can only play catcher, 4 players who can only play first base, 6 players who can only pitch, and 14 players who can play in any of the remaining 6 positions? (Lesson 0-7)

Find the values for x and y that make each matrix equation true. (Lesson 0-6)

96. $\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 4x - 3 \\ y - 2 \end{bmatrix}$

97. $\begin{bmatrix} 3y \\ 10 \end{bmatrix} = \begin{bmatrix} 27 + 6x \\ 5y \end{bmatrix}$

98. $[9 \quad 11] = [3x + 3y \quad 2x + 1]$

Use any method to solve the system of equations. State whether the system is *consistent*, *dependent*, *independent*, or *inconsistent*. (Lesson 0-5)

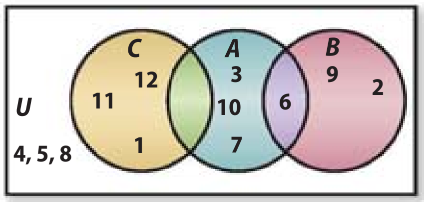
99. $2x + 3y = 36$
$4x + 2y = 48$

100. $5x + y = 25$
$10x + 2y = 50$

101. $7x + 8y = 30$
$7x + 16y = 46$

102. BUSINESS A used book store sells 1400 paperback books per week at \$2.25 per book. The owner estimates that he will sell 100 fewer books for each \$0.25 increase in price. What price will maximize the income of the store? (Lesson 0-3)

Use the Venn diagram to find each of the following. (Lesson 0-1)



103. A'

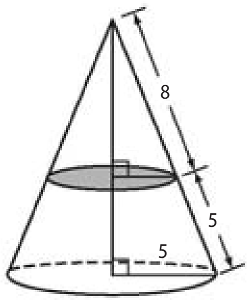
104. $A \cup B$

105. $B \cap C$

106. $A \cap B$

Skills Review for Standardized Tests

107. SAT/ACT A circular cone with a base of radius 5 has been cut as shown in the figure.



What is the height of the smaller top cone?

A $\frac{8}{13}$

B $\frac{96}{13}$

C $\frac{96}{12}$

D $\frac{96}{5}$

E $\frac{104}{5}$

108. REVIEW Which function is linear?

F $f(x) = x^2$

G $g(x) = 2.7$

H $f(x) = \sqrt{9 - x^2}$

J $g(x) = \sqrt{x - 1}$

109. Louis is flying from Denver to Dallas for a convention. He can park his car in the Denver airport long-term lot or in the nearby shuttle parking facility. The long-term lot costs \$1 per hour or any fraction thereof with a maximum charge of \$6 per day. In the shuttle facility, he has to pay \$4 for each day or part of a day. Which parking lot is less expensive if Louis returns after 2 days and 3 hours?

A shuttle facility

B airport lot

C They will both cost the same.

D cannot be determined with the information given

110. REVIEW Given $y = 2.24x + 16.45$, which statement best describes the effect of moving the graph down two units?

F The y-intercept increases.

G The x-intercept remains the same.

H The x-intercept increases.

J The y-intercept remains the same.

LESSON 1-2 Analyzing Graphs of Functions and Relations

Then

- You identified functions. (Lesson 1-1)

Now

1. Use graphs of functions to estimate function values and find domains, ranges, *y*-intercepts, and zeros of functions.
2. Explore symmetries of graphs, and identify even and odd functions.

Why?

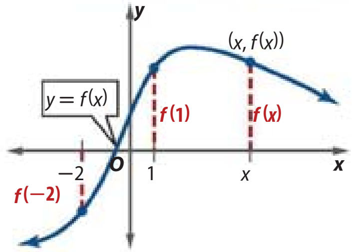
- With more people turning to the Internet for news and entertainment, Internet advertising is big business. The total revenue R in millions of dollars earned by U.S. companies from Internet advertising from 1999 to 2008 can be approximated by $R(t) = 17.7t^3 - 269t^2 + 1458t - 910$, $1 \le t \le 10$, where t represents the number of years since 1998. Graphs of functions like this can help you visualize relationships between real-world quantities.

NewVocabulary

zeros
roots
line symmetry
point symmetry
even function
odd function

1 Analyzing Function Graphs

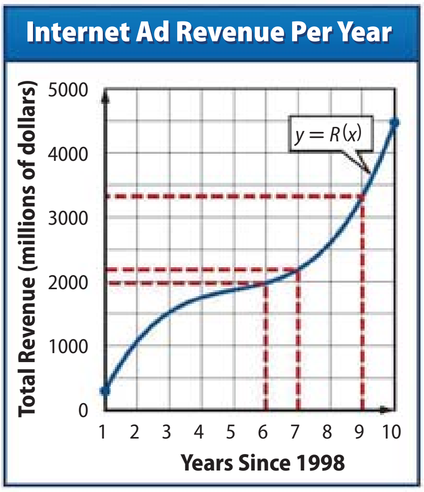
The graph of a function f is the set of ordered pairs $(x, f(x))$ such that x is in the domain of f. In other words, the graph of f is the graph of the equation $y = f(x)$. So, the value of the function is the directed distance y of the graph from the point x on the x-axis as shown.



You can use a graph to estimate function values.

Real-World Example 1 Estimate Function Values

INTERNET **Consider the graph of function R shown.**



a. Use the graph to estimate total Internet advertising revenue in 2007. Confirm the estimate algebraically.

The year 2007 is 9 years after 1998. The function value at $x = 9$ appears to be about \$3300 million, so the total Internet advertising revenue in 2007 was about \$3.3 billion.

To confirm this estimate algebraically, find $f(9)$.

$$f(9) = 17.7(9)^3 - 269(9)^2 + 1458(9) - 910$$
$$\approx 3326.3 \text{ million or } 3.326 \text{ billion}$$

Therefore, the graphical estimate of \$3.3 billion is reasonable.

b. Use the graph to estimate the year in which total Internet advertising revenue reached \$2 billion. Confirm the estimate algebraically.

The value of the function appears to reach \$2 billion or \$2000 million for x-values between 6 and 7. So, the total revenue was nearly \$2 billion in 1998 + 6 or 2004 but had exceeded \$2 billion by the end of 1998 + 7 or 2005.

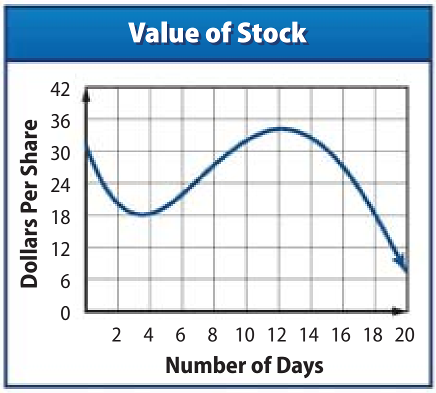
To confirm algebraically, find $f(6)$ and $f(7)$.

$$f(6) = 17.7(6)^3 - 269(6)^2 + 1458(6) - 910 \text{ or about } 1977 \text{ million}$$
$$f(7) = 17.7(7)^3 - 269(7)^2 + 1458(7) - 910 \text{ or about } 2186 \text{ million}$$

In billions, $f(6) \approx 1.977$ billion and $f(7) \approx 2.186$ billion. Therefore, the graphical estimate that total Internet advertising revenue reached \$2 billion in 2005 is reasonable.

GuidedPractice

1. **STOCKS** An investor assessed the average daily value of a share of a certain stock over a 20-day period. The value of the stock can be approximated by $v(d) = 0.002d^4 - 0.11d^3 + 1.77d^2 - 8.6d + 31$, $0 \leq d \leq 20$, where d represents the day of the assessment.



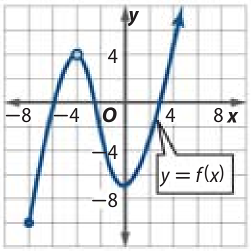
A. Use the graph to estimate the value of the stock on the 10th day. Confirm your estimate algebraically.

B. Use the graph to estimate the days during which the stock was valued at \$30 per share. Confirm your estimate algebraically.

You can also use a graph to find the domain and range of a function. Unless the graph of a function is bounded on the left by a circle or a dot, you can assume that the function extends beyond the edges of the graph.

Example 2 Find Domain and Range

Use the graph of f to find the domain and range of the function.



Domain

- The dot at $(-8, -10)$ indicates that the domain of f starts at and includes -8.
- The circle at $(-4, 4)$ indicates that -4 is not part of the domain.
- The arrow on the right side indicates that the graph will continue without bound.

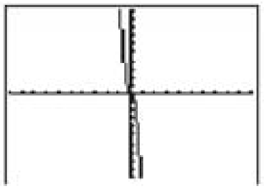
The domain of f is $[-8, -4) \cup (-4, \infty)$. In set-builder notation, the domain is $\{x \mid -8 \leq x, x \neq -4, x \in \mathbb{R}\}$.

Range

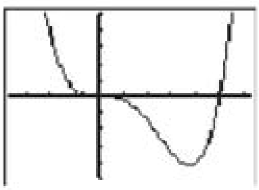
The graph does not extend below $f(-8)$ or -10, but $f(x)$ increases without bound for greater and greater values of x. So, the range of f is $[-10, \infty)$.

TechnologyTip

Choosing an Appropriate Window The viewing window of a graph is a picture of the graph for a specific domain and range. This may not represent the entire graph. Notice the difference in the graphs of $f(x) = x^4 - 20x^3$ shown below.



[−10, 10] scl: 1 by [−10, 10] scl: 1

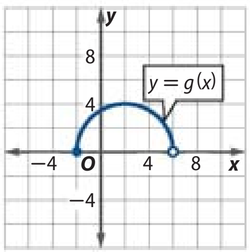


[−15, 25] scl: 4 by [−20,000, 20,000] scl: 4000

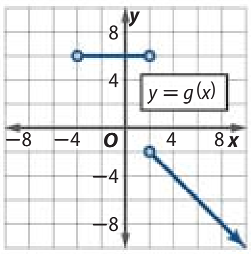
GuidedPractice

Use the graph of g to find the domain and range of each function.

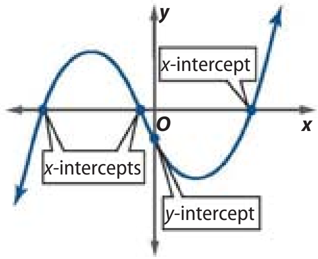
2A.



2B.



A point where a graph intersects or meets the x- or y-axis is called an intercept. An x-intercept of a graph occurs where $y = 0$. A y-intercept of a graph occurs where $x = 0$. The graph of a function can have 0, 1, or more x-intercepts, but at most one y-intercept.

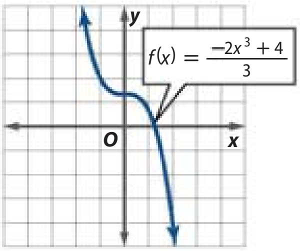


To find the y-intercept of a graph of a function f algebraically, find $f(0)$.

Example 3 Find y-Intercepts

Use the graph of each function to approximate its y-intercept. Then find the y-intercept algebraically.

a.



Estimate Graphically

It appears that $f(x)$ intersects the y-axis at approximately $\left(0, 1\frac{1}{3}\right)$, so the y-intercept is about $1\frac{1}{3}$.

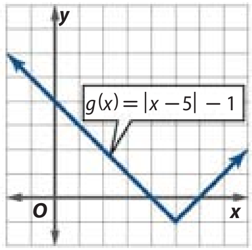
Solve Algebraically

Find $f(0)$.

$$f(0) = \frac{-2(0)^3 + 4}{3} \text{ or } \frac{4}{3}$$

The y-intercept is $\frac{4}{3}$ or $1\frac{1}{3}$.

b.



Estimate Graphically

It appears that $g(x)$ intersects the y-axis at (0, 4), so the y-intercept is 4.

Solve Algebraically

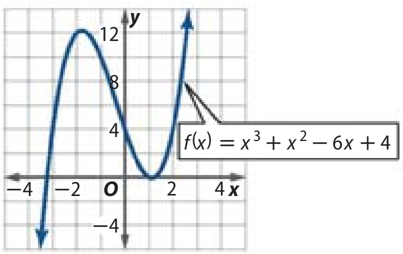
Find $g(0)$.

$$g(0) = |0 - 5| - 1 \text{ or } 4$$

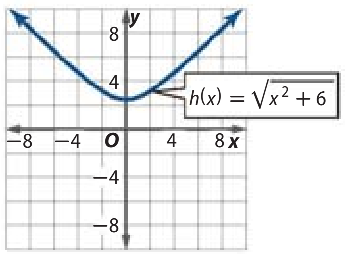
The y-intercept is 4.

GuidedPractice

3A.



3B.



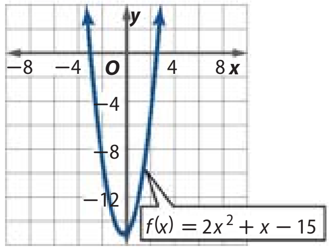
StudyTip

Labeling Axis on Graphs When you label an axis on the graph, the variable letter for the domain is on the x-axis and the variable letter for the range is on the y-axis. Throughout this book, there are many different variables used for both the domain and range. For consistency, the horizontal axis is always x and the vertical axis is always y.

The x-intercepts of the graph of a function are also called the **zeros** of a function. The solutions of the corresponding equation are called the **roots** of the equation. To find the zeros of a function f, set the function equal to 0 and solve for the independent variable.

Example 4 Find Zeros

Use the graph of $f(x) = 2x^2 + x - 15$ to approximate its zero(s). Then find its zero(s) algebraically.



Estimate Graphically

The x-intercepts appear to be at about -3 and 2.5.

Solve Algebraically

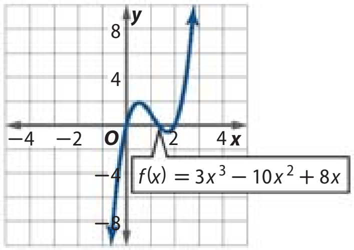
$2x^2 + x - 15 = 0$		Let $f(x) = 0$.
$(2x - 5)(x + 3) = 0$		Factor.
$2x - 5 = 0$ or $x + 3 = 0$		Zero Product Property
$x = 2.5$	$x = -3$	Solve for x.

The zeros of f are -3 and 2.5.

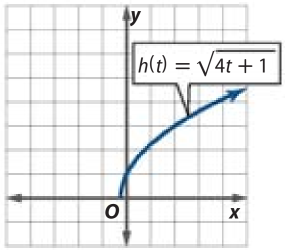
GuidedPractice

Use the graph of each function to approximate its zero(s). Then find its zero(s) algebraically.

4A.



4B.



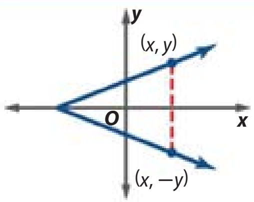
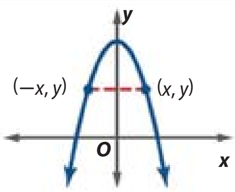
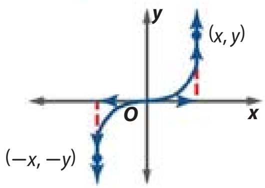
2 Symmetry of Graphs

Graphs of relations can have two different types of symmetry. Graphs with **line symmetry** can be folded along a line so that the two halves match exactly. Graphs that have **point symmetry** can be rotated 180° with respect to a point and appear unchanged. The three most common types of symmetry are shown below.

StudyTip

Symmetry, Relations, and Functions There are numerous *relations* that have x-axis, y-axis, and origin symmetry. However, the only *function* that has all three types of symmetry is the zero function, $f(x) = 0$.

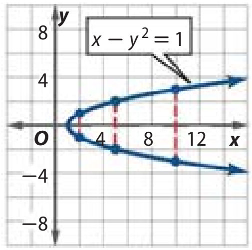
KeyConcept Tests for Symmetry

Graphical Test	Model	Algebraic Test
The graph of a relation is *symmetric with respect to the x-axis* if and only if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.		Replacing y with $-y$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the y-axis* if and only if for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.		Replacing x with $-x$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the origin* if and only if for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.		Replacing x with $-x$ and y with $-y$ produces an equivalent equation.

Example 5 Test for Symmetry

Use the graph of each equation to test for symmetry with respect to the x-axis, y-axis, and the origin. Support the answer numerically. Then confirm algebraically.

a. $x - y^2 = 1$



Analyze Graphically

The graph appears to be symmetric with respect to the x-axis because for every point (x, y) on the graph, there is a point $(x, -y)$.

Support Numerically

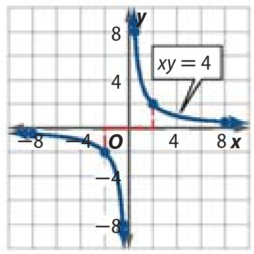
A table of values supports this conjecture.

x	2	2	5	5	10	10
y	1	−1	2	−2	3	−3
(x, y)	(2, 1)	(2, −1)	(5, 2)	(5, −2)	(10, 3)	(10, −3)

Confirm Algebraically

Because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$, the graph is symmetric with respect to the x-axis.

b. $xy = 4$



Analyze Graphically

The graph appears to be symmetric with respect to the origin because for every point (x, y) on the graph, there is a point $(-x, -y)$.

Support Numerically

A table of values supports this conjecture.

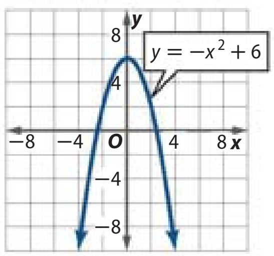
x	−8	−2	−0.5	0.5	2	8
y	−0.5	−2	−8	8	2	0.5
(x, y)	(−8, −0.5)	(−2, −2)	(−0.5, −8)	(0.5, 8)	(2, 2)	(8, 0.5)

Confirm Algebraically

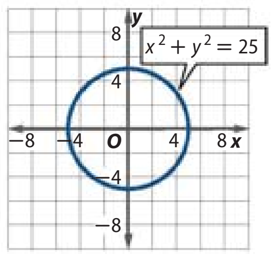
Because $(-x)(-y) = 4$ is equivalent to $xy = 4$, the graph is symmetric with respect to the origin.

Guided Practice

5A.



5B.



Graphs of functions can have y-axis or origin symmetry. Functions with these types of symmetry have special names.

KeyConcept Even and Odd Functions

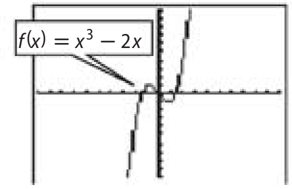
Type of Function	Algebraic Test
Functions that are symmetric with respect to the y-axis are called **even functions**.	For every x in the domain of f, $f(-x) = f(x)$.
Functions that are symmetric with respect to the origin are called **odd functions**.	For every x in the domain of f, $f(-x) = -f(x)$.

Example 6 Identify Even and Odd Functions

GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.**

a. $f(x) = x^3 - 2x$

It appears that the graph of the function is symmetric with respect to the origin. Test this conjecture.



[−10, 10] scl: 1 by [−10, 10] scl: 1

$f(-x) = (-x)^3 - 2(-x)$ Substitute $-x$ for x.

$= -x^3 + 2x$ Simplify.

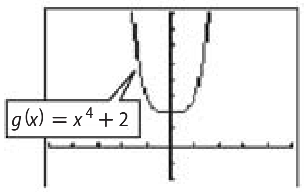
$= -(x^3 - 2x)$ Distributive Property

$= -f(x)$ Original function $f(x) = x^3 - 2x$

The function is odd because $f(-x) = -f(x)$. Therefore, the function is symmetric with respect to the origin.

b. $g(x) = x^4 + 2$

It appears that the graph of the function is symmetric with respect to the y-axis. Test this conjecture.



[−5, 5] scl: 1 by [−2, 8] scl: 1

$g(-x) = (-x)^4 + 2$ Substitute $-x$ for x.

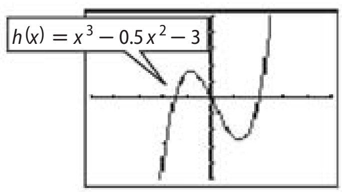
$= x^4 + 2$ Simplify.

$= g(x)$ Original function $g(x) = x^4 + 2$

The function is even because $g(-x) = g(x)$. Therefore, the function is symmetric with respect to the y-axis.

c. $h(x) = x^3 - 0.5x^2 - 3x$

It appears that the graph of the function may be symmetric with respect to the origin. Test this conjecture algebraically.



[−5, 5] scl: 1 by [−5, 5] scl: 1

$h(-x) = (-x)^3 - 0.5(-x)^2 - 3(-x)$ Substitute $-x$ for x.

$= -x^3 - 0.5x^2 + 3x$ Simplify.

Because $-h(x) = -x^3 + 0.5x^2 + 3x$, the function is neither even nor odd because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$.

StudyTip

Even and Odd Functions It is important to always confirm symmetry algebraically. Graphs that appear to be symmetrical may not actually be.

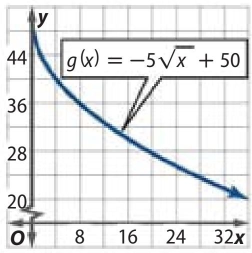
GuidedPractice

6A. $f(x) = \frac{2}{x^2}$ **6B.** $g(x) = 4\sqrt{x}$ **6C.** $h(x) = x^5 - 2x^3 + x$

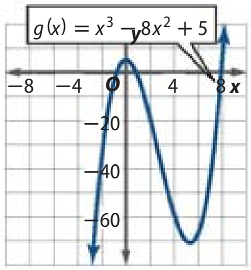
Exercises

= Step-by-Step Solutions begin on page R29.

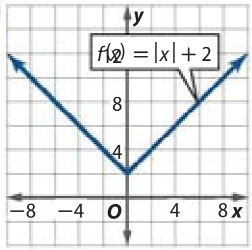
Use the graph of each function to estimate the indicated function values. Then confirm the estimate algebraically. Round to the nearest hundredth, if necessary. (Example 1)

1. 

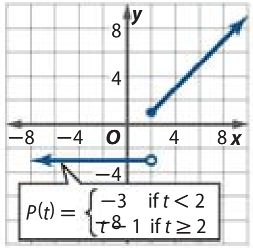
a. $g(6)$ **b.** $g(12)$ **c.** $g(19)$

2. 

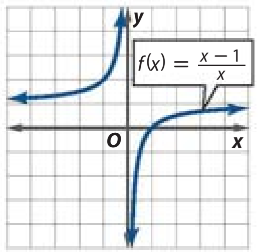
a. $g(-2)$ **b.** $g(1)$ **c.** $g(8)$

3. 

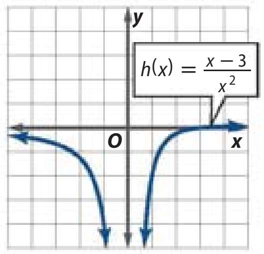
a. $f(-8)$ **b.** $f(-3)$ **c.** $f(0)$

4. 

a. $P(-6)$ **b.** $P(2)$ **c.** $P(9)$

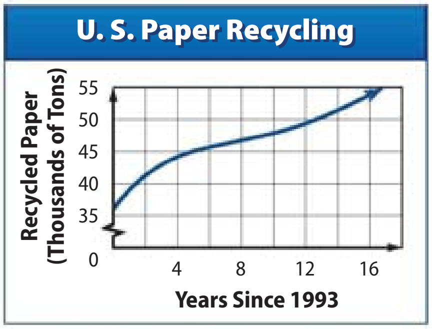
5. 

a. $f(-3)$ **b.** $f(0.5)$ **c.** $f(0)$

6. 

a. $h(-1)$ **b.** $h(1.5)$ **c.** $h(2)$

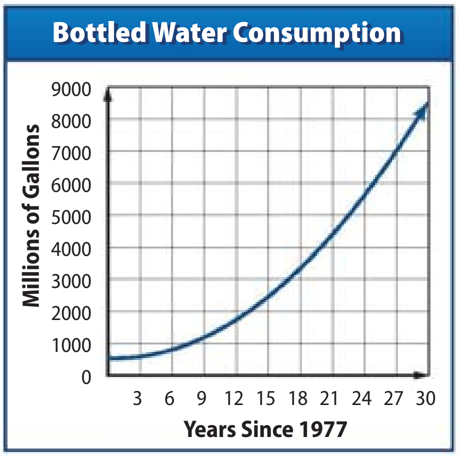
7. **RECYLING** The quantity of paper recycled in the United States in thousands of tons from 1993 to 2007 can be modeled by $p(x) = -0.0013x^4 + 0.0513x^3 - 0.662x^2 + 4.128x + 35.75$, where x is the number of years since 1993. (Example 1)



a. Use the graph to estimate the amount of paper recycled in 1993, 1999, and 2006. Then find each value algebraically.

b. Use the graph to estimate the year in which the quantity of paper recycled reached 50,000 tons. Confirm algebraically.

8. **WATER** Bottled water consumption from 1977 to 2006 can be modeled using $f(x) = 9.35x^2 - 12.7x + 541.7$, where x represents the number of years since 1977. (Example 1)

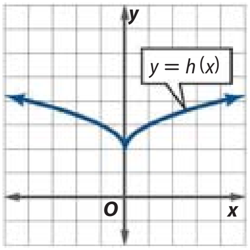


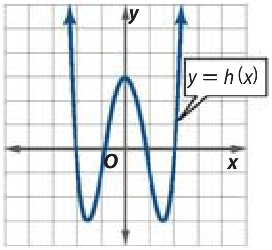
a. Use the graph to estimate the amount of bottled water consumed in 1994.

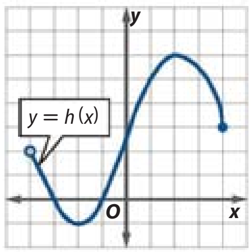
b. Find the 1994 consumption algebraically. Round to the nearest ten million gallons.

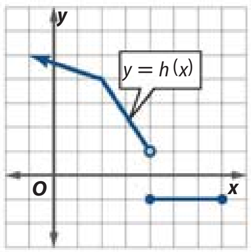
c. Use the graph to estimate when water consumption was 6 billion gallons. Confirm algebraically.

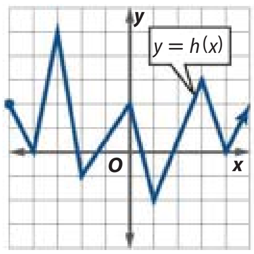
Use the graph of h to find the domain and range of each function. (Example 2)

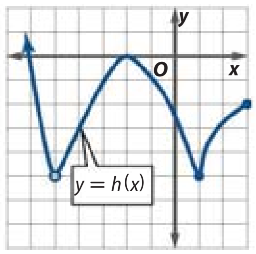
9. 

10. 

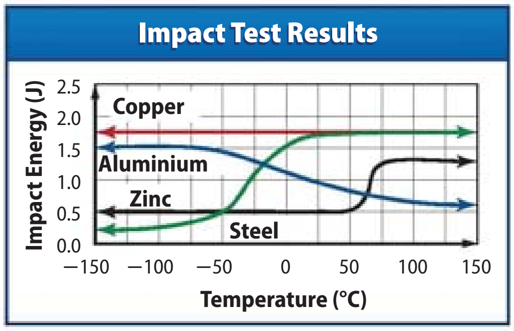
11. 

12. 

13. 

14. 

15. **ENGINEERING** Tests on the physical behavior of four metal specimens are performed at various temperatures in degrees Celsius. The impact energy, or energy absorbed by the sample during the test, is measured in joules. The test results are shown. (Example 2)

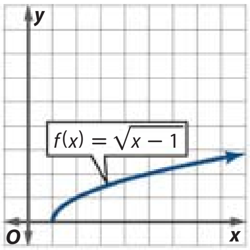


a. State the domain and range of each function.

b. Use the graph to estimate the impact energy of each metal at 0°C.

Use the graph of each function to find its y-intercept and zero(s). Then find these values algebraically. (Examples 3 and 4)

16.



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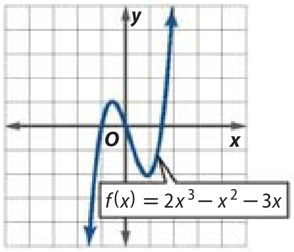

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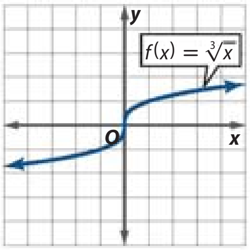

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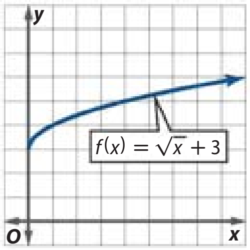

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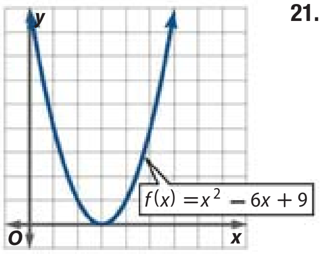

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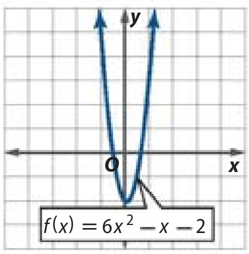

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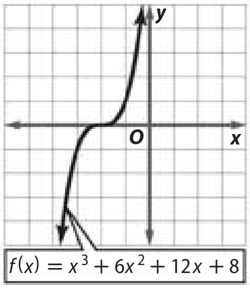
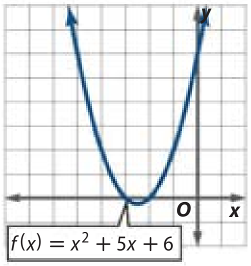
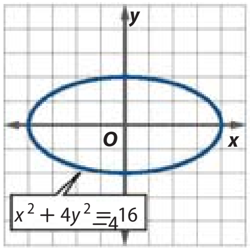


23.



Use the graph of each equation to test for symmetry with respect to the x-axis, y-axis, and the origin. Support the answer numerically. Then confirm algebraically. (Example 5)

24.



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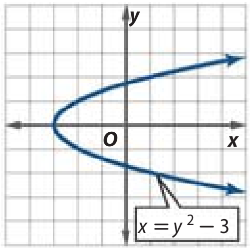

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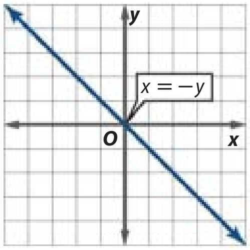

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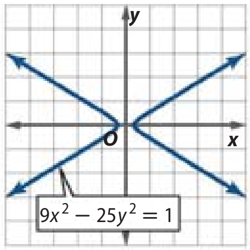

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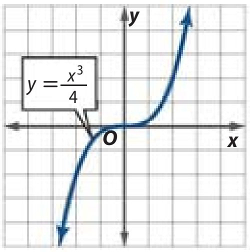

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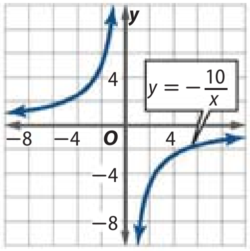
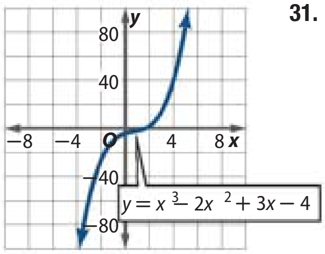
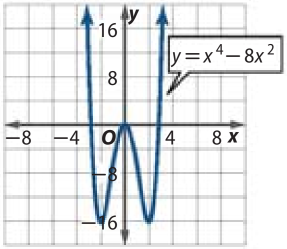


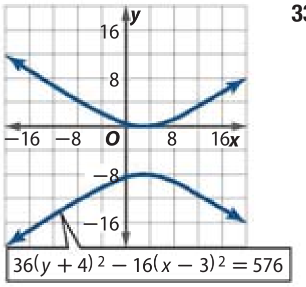
30.



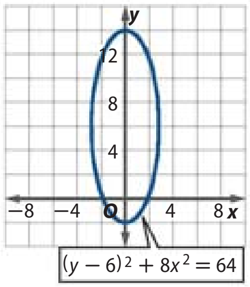
31.



32.



33.



GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.** (Example 6)

34. $f(x) = x^2 + 6x + 10$

35. $f(x) = -2x^3 + 5x - 4$

36. $g(x) = \sqrt{x + 6}$

37. $h(x) = \sqrt{x^2 - 9}$

38. $h(x) = |8 - 2x|$

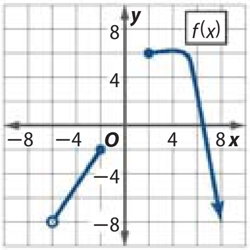
39. $f(x) = |x^3|$

40. $f(x) = \dfrac{x + 4}{x - 2}$

41. $g(x) = \dfrac{x^2}{x + 1}$

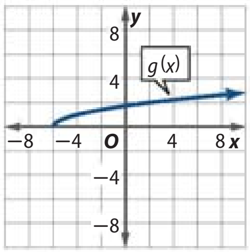
Use the graph of each function to estimate the indicated function values.

42.



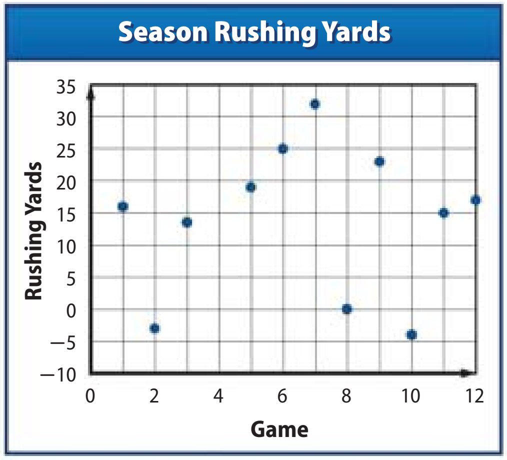
a. $f(-2)$ b. $f(-6)$ c. $f(0)$

43.



a. $g(-8)$ b. $g(-6)$ c. $g(-2)$

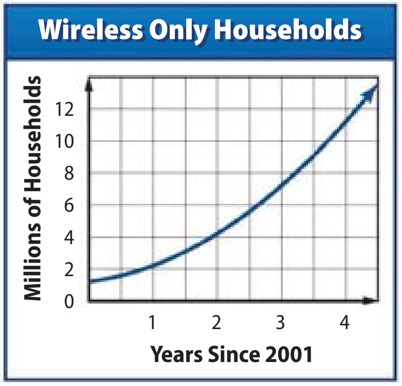
44. FOOTBALL A running back's rushing yards for each game in a season are shown.



a. State the domain and range of the relation.

b. In what game did the player rush for no yards?

45 PHONES The number of households h in millions with only wireless phone service from 2001 to 2005 can be modeled by $h(x) = 0.5x^2 + 0.5x + 1.2$, where x represents the number of years after 2001.



a. State the relevant domain and approximate the range.

b. Use the graph to estimate the number of households with only wireless phone service in 2003. Then find it algebraically.

c. Use the graph to approximate the y-intercept of the function. Then find it algebraically. What does the y-intercept represent?

d. Does this function have any zeros? If so, estimate them and explain their meaning. If not, explain why.

46. FUNCTIONS Consider $f(x) = x^n$.

a. Use a graphing calculator to graph $f(x)$ for values of n in the range $1 \leq n \leq 6$, where $n \in \mathbb{N}$.

b. Describe the domain and range of each function.

c. Describe the symmetry of each function.

d. Predict the domain, range, and symmetry for $f(x) = x^{35}$. Explain your reasoning.

47. PHARMACOLOGY Suppose the number of milligrams of a pain reliever in the bloodstream x hours after taking a dose is modeled by $f(x) = 0.5x^4 + 3.45x^3 - 96.65x^2 + 347.7x$.

a. Use a graphing calculator to graph the function.

b. State the relevant domain. Explain your reasoning.

c. What was the approximate maximum amount of pain reliever, in milligrams, in the bloodstream at any given time?

GRAPHING CALCULATOR Graph each function and locate the zeros for each function. Confirm your answers algebraically.

48. $f(x) = \dfrac{4x - 1}{x}$

49. $f(x) = \dfrac{x^2 + 9}{x + 3}$

50. $h(x) = \sqrt{x^2 + 4x + 3}$

51. $h(x) = 2\sqrt{x + 12} - 8$

52. $g(x) = -12 + \dfrac{4}{x}$

53. $g(x) = \dfrac{6}{x} + 3$

54. TELEVISION In a city, the percent of households h with basic cable for the years 1986 through 2012 can be modeled using $h(x) = -0.115x^2 + 4.43x + 25.6$, $0 \leq x \leq 26$, where x represents the number of years after 1986.

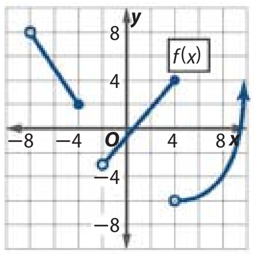
a. Use a graphing calculator to graph the function.

b. What percent of households had basic cable in 2005? Round to the nearest percent.

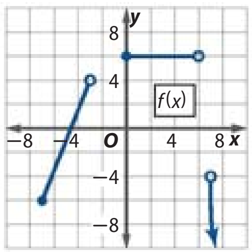
c. For what years was the percent of subscribers greater than 65%?

Use the graph of f to find the domain and range of each function.

55.



56.



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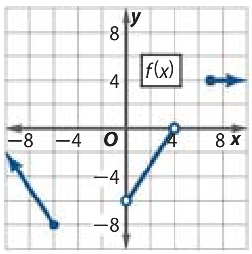

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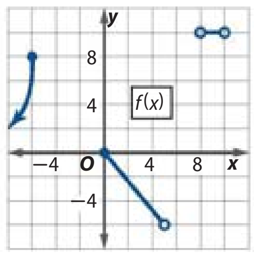
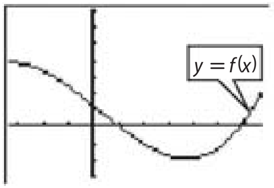


59. **POPULATION** At the beginning of 1900 a city had a population of 140,000. The percent change from 1930 to 2010 is modeled by $f(x) = 0.0001x^3 - 0.001x^2 - 0.825x + 12.58$, where x is the number of years since 1930.



[−50, 100] scl: 15 by [−30, 70] scl: 10

a. State the relevant domain and estimate the range for this domain.

b. Use the graph to approximate the y-intercept. Then find the y-intercept algebraically. What does the y-intercept represent?

c. Find and interpret the zeros of the function.

d. Use the model to determine what the percent population change will be in 2080. Does this value seem realistic? Explain your reasoning.

60. **STOCK MARKET** The percent p a stock price has fluctuated in one year can be modeled by $p(x) = 0.0005x^4 - 0.0193x^3 + 0.243x^2 - 1.014x + 1.04$, where x is the number of months since January.

a. Use a graphing calculator to graph the function.

b. State the relevant domain and estimate the range.

c. Use the graph to approximate the y-intercept. Then find the y-intercept algebraically. What does the y-intercept represent?

d. Find and interpret any zeros of the function.

61. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the range values of $f(x) = \frac{1}{x-2}$ as x approaches 2.

a. **TABULAR** Copy and complete the table below. Add an additional value to the left and right of 2.

x	1.99	1.999	2	2.001	2.01
$f(x)$					

b. **ANALYTICAL** Use the table from part **a** to describe the behavior of the function as x approaches 2.

c. **GRAPHICAL** Graph the function. Does the graph support your conjecture from part **b**? Explain.

d. **VERBAL** Make a conjecture as to why the graph of the function approaches the value(s) found in part **c**, and explain any inconsistencies present in the graph.

GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.**

62. $f(x) = x^2 - x - 6$

63. $g(n) = n^2 - 37$

64. $h(x) = x^6 + 4$

65. $f(g) = g^9$

66. $g(y) = y^4 + 8y^2 + 81$

67. $h(y) = y^5 - 17y^3 + 16y$

68. $h(b) = b^4 - 2b^3 - 13b^2 + 14b + 24$

H.O.T. Problems Use Higher-Order Thinking Skills

OPEN ENDED **Sketch a graph that matches each description.**

69. passes through (−3, 8), (−4, 4), (−5, 2), and (−8, 1) and is symmetric with respect to the y-axis

70. passes through (0, 0), (2, 6), (3, 12), and (4, 24) and is symmetric with respect to the x-axis

71 passes through (−3, −18), (−2, −9), and (−1, −3) and is symmetric with respect to the origin

72. passes through (4, −16), (6, −12), and (8, −8) and represents an even function

73. **WRITING IN MATH** Explain why a function can have 0, 1, or more x-intercepts but only one y-intercept.

74. **CHALLENGE** Use a graphing calculator to graph $f(x) = \frac{2x^2 + 3x - 2}{x^3 - 4x^2 - 12x}$, and predict its domain. Then confirm the domain algebraically. Explain your reasoning.

REASONING **Determine whether each statement is *true* or *false*. Explain your reasoning.**

75. The range of $f(x) = nx^2$, where n is any integer, is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

76. The range of $f(x) = \sqrt{nx}$, where n is any integer, is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

77. All odd functions are also symmetric with respect to the line $y = -x$.

78. An even function rotated $180n°$ about the origin, where n is any integer, remains an even function.

REASONING **If $a(x)$ is an odd function, determine whether $b(x)$ is *odd*, *even*, *neither*, or *cannot be determined*. Explain your reasoning.**

79. $b(x) = a(-x)$

80. $b(x) = -a(x)$

81. $b(x) = [a(x)]^2$

82. $b(x) = a(|x|)$

83. $b(x) = [a(x)]^3$

REASONING **State whether a graph with each type of symmetry *always*, *sometimes*, or *never* represents a function. Explain your reasoning.**

84. symmetric with respect to the line $x = 4$

85. symmetric with respect to the line $y = 2$

86. symmetric with respect to the line $y = x$

87. symmetric with respect to both the x- and y-axes

88. **WRITING IN MATH** Can a function be both even and odd? Explain your reasoning.

Spiral Review

Find each function value. (Lesson 1-1)

89. $g(x) = x^2 - 10x + 3$

a. $g(2)$

b. $g(-4x)$

c. $g(1 + 3n)$

90. $h(x) = 2x^2 + 4x - 7$

a. $h(-9)$

b. $h(3x)$

c. $h(2 + m)$

91. $p(x) = \dfrac{2x^3 + 2}{x^2 - 2}$

a. $p(3)$

b. $p(x^2)$

c. $p(x + 1)$

92. GRADES The midterm grades for a Chemistry class of 25 students are shown. Find the measures of spread for the data set. (Lesson 0-8)

Midterm Grades				
89	76	91	72	81
81	65	74	80	74
73	92	76	83	96
66	61	80	74	70
97	78	73	62	72

93. PLAYING CARDS From a standard 52-card deck, find how many 5-card hands are possible that fit each description. (Lesson 0-7)

a. 3 hearts and 2 clubs

b. 1 ace, 2 jacks, and 2 kings

c. all face cards

Find the following for $A = \begin{bmatrix} -6 & 3 \\ -5 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -7 \\ 2 & -3 \end{bmatrix}$, **and** $C = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. (Lesson 0-6)

94. $4A - 2B$

95. $3C + 2A$

96. $-2(B - 3A)$

Evaluate each expression. (Lesson 0-4)

97. $27^{\frac{1}{3}}$

98. $64^{\frac{5}{6}}$

99. $49^{-\frac{1}{2}}$

100. $16^{-\frac{3}{4}}$

101. $25^{\frac{3}{2}}$

102. $36^{-\frac{3}{2}}$

103. GENETICS Suppose R and W represent two genes that a plant can inherit from its parents. The terms of the expansion of $(R + W)^2$ represent the possible pairings of the genes in the offspring. Write $(R + W)^2$ as a polynomial. (Lesson 0-3)

Simplify. (Lesson 0-2)

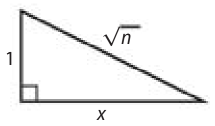
104. $(2 + i)(4 + 3i)$

105. $(1 + 4i)^2$

106. $(2 - i)(3 + 2i)(1 - 4i)$

Skills Review for Standardized Tests

107. SAT/ACT In the figure, if n is a real number greater than 1, what is the value of x in terms of n?



A $\sqrt{n^2 - 1}$

B $\sqrt{n - 1}$

C $\sqrt{n + 1}$

D $n - 1$

E $n + 1$

108. REVIEW Which inequality describes the range of $f(x) = x^2 + 1$ over the domain $-2 < x < 3$?

F $5 \leq y < 9$

G $2 < y < 10$

H $1 < y < 9$

J $1 \leq y < 10$

109. Which of the following is an even function?

A $f(x) = 2x^4 + 6x^3 - 5x^2 - 8$

B $g(x) = 3x^6 + x^4 - 5x^2 + 15$

C $m(x) = x^4 + 3x^3 + x^2 + 35x$

D $h(x) = 4x^6 + 2x^4 + 6x - 4$

110. Which of the following is the domain of $g(x) = \dfrac{1 + x}{x^2 - 16x}$?

F $(-\infty, 0) \cup (0, 16) \cup (16, \infty)$

G $(-\infty, 0] \cup [16, \infty)$

H $(-\infty, -1) \cup (-1, \infty)$

J $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

LESSON 1-3 Continuity, End Behavior, and Limits

Then

- You found domain and range using the graph of a function. (Lesson 1-2)

Now

1. Use limits to determine the continuity of a function, and apply the Intermediate Value Theorem to continuous functions.
2. Use limits to describe end behavior of functions.

Why?

- Since the early 1980s, the current minimum wage has jumped up several times. The graph of the minimum wage as a function of time shows these jumps as breaks in the graph, such as those at $x = 1990$, $x = 1996$, and $x = 2008$.

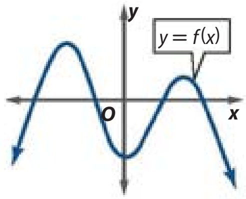
NewVocabulary

continuous function
limit
discontinuous function
infinite discontinuity
jump discontinuity
removable discontinuity
nonremovable discontinuity
end behavior

1 Continuity

The graph of a **continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

One condition for a function $f(x)$ to be continuous at $x = c$ is that the function must approach a unique function value as x-values approach c from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called a **limit**.

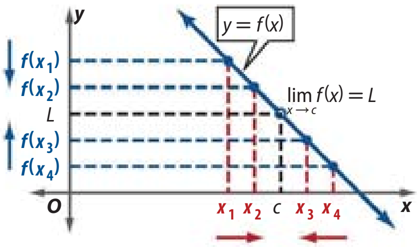


$f(x)$ is continuous for all x.

KeyConcept Limits

Words If the value of $f(x)$ approaches a unique value L as x approaches c from each side, then the limit of $f(x)$ as x approaches c is L.

Symbols $\lim_{x \to c} f(x) = L$, which is read *The limit of $f(x)$ as x approaches c is L.*

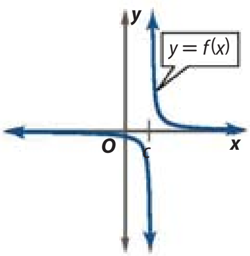


To understand what it means for a function to be continuous from an algebraic perspective, it helps to examine the graphs of **discontinuous functions**, or functions that are not continuous. Functions can have many different types of discontinuity.

KeyConcept Types of Discontinuity

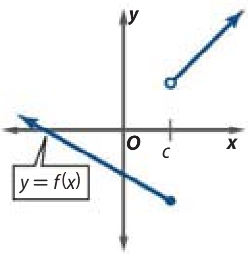
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example



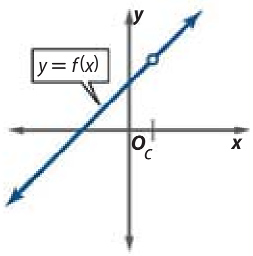
A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

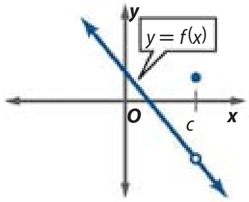
Example



StudyTip

Limits Whether $f(x)$ exists at $x = c$ has no bearing on the existence of the *limit* of $f(x)$ as x approaches c.

Notice that for graphs of functions with a removable discontinuity, the limit of $f(x)$ at point c exists, but either the value of the function at c is undefined, or, as with the graph shown, the value of $f(c)$ is not the same as the value of the limit at point c.



Infinite and jump discontinuities are classified as **nonremovable discontinuities**. A nonremovable discontinuity cannot be eliminated by redefining the function at that point, since the function approaches different values from the left and right sides at that point or does not approach a single value at all. Instead it is increasing or decreasing indefinitely.

These observations lead to the following test for the continuity of a function.

ConceptSummary Continuity Test

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

- $f(x)$ is defined at c. That is, $f(c)$ exists.
- $f(x)$ approaches the same value from either side of c. That is, $\lim_{x \to c} f(x)$ exists.
- The value that $f(x)$ approaches from each side of c is $f(c)$. That is, $\lim_{x \to c} f(x) = f(c)$.

Example 1 Identify a Point of Continuity

Determine whether $f(x) = 2x^2 - 3x - 1$ is continuous at $x = 2$. Justify using the continuity test.

Check the three conditions in the continuity test.

1. **Does $f(2)$ exist?**

   Because $f(2) = 1$, the function is defined at $x = 2$.

2. **Does $\lim_{x \to 2} f(x)$ exist?**

   Construct a table that shows values of $f(x)$ for x-values approaching 2 from the left and from the right.

	x approaches 2 →				← *x* approaches 2		
x	1.9	1.99	1.999	**2.0**	2.001	2.01	2.1
$f(x)$	0.52	0.95	0.995		1.005	1.05	1.52

   The pattern of outputs suggests that as the value of x gets closer to 2 from the left and from the right, $f(x)$ gets closer to 1. So, we estimate that $\lim_{x \to 2} f(x) = 1$.

3. **Does $\lim_{x \to 2} f(x) = f(2)$?**

   Because $\lim_{x \to 2} (2x^2 - 3x - 1)$ is estimated to be 1 and $f(2) = 1$, we conclude that $f(x)$ is continuous at $x = 2$. The graph of $f(x)$ shown in Figure 1.3.1 supports this conclusion.

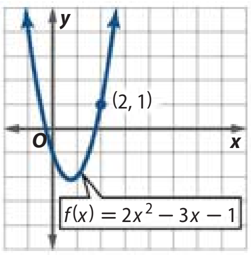


Figure 1.3.1

GuidedPractice

Determine whether each function is continuous at $x = 0$. Justify using the continuity test.

1A. $f(x) = x^3$

1B. $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

If just one of the conditions for continuity is not satisfied, the function is discontinuous at $x = c$. Examining a function can help you identify the type of discontinuity at that point.

Example 2 Identify a Point of Discontinuity

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

a. $f(x) = \begin{cases} 3x - 2 & \text{if } x > -3 \\ 2 - x & \text{if } x \leq -3 \end{cases}$; **at $x = -3$**

1. Because $f(-3) = 5$, $f(-3)$ exists.

2. Investigate function values close to $f(-3)$.

x approaches -3 → ← x approaches -3

x	−3.1	−3.01	−3.001	−3.0	−2.999	−2.99	−2.9
$f(x)$	5.1	5.01	5.001		−10.997	−10.97	10.7

The pattern of outputs suggests that $f(x)$ approaches 5 as x approaches -3 from the left and -11 as $f(x)$ approaches -3 from the right. Because these values are not the same, $\lim_{x \to -3} f(x)$ does not exist. Therefore, $f(x)$ is discontinuous at $x = -3$. Because $f(x)$ approaches two different values when $x = -3$, $f(x)$ has a jump discontinuity at $x = -3$. The graph of $f(x)$ in Figure 1.3.2 supports this conclusion.

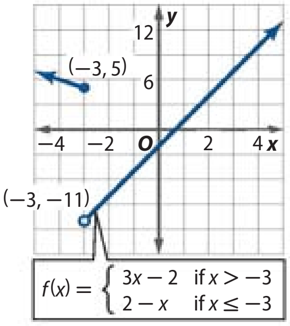


Figure 1.3.2

b. $f(x) = \dfrac{x + 3}{x^2 - 9}$; **at $x = -3$ and $x = 3$**

1. Because $f(-3) = \frac{0}{0}$ and $f(3) = \frac{6}{0}$, both of which are undefined, $f(-3)$ and $f(3)$ do not exist. Therefore, $f(x)$ is discontinuous at both $x = -3$ and at $x = 3$.

2. Investigate function values close to $f(-3)$.

x approaches -3 → ← x approaches -3

x	−3.1	−3.01	−3.001	−3.0	−2.999	−2.99	−2.9
$f(x)$	−0.164	−0.166	−0.167		−0.167	−0.167	−0.169

The pattern of outputs suggests that $f(x)$ approaches a limit close to -0.167 as x approaches -3 from each side, so $\lim_{x \to -3} f(x) \approx -0.167$ or $-\frac{1}{6}$.

Investigate function values close to $f(3)$.

x approaches 3 → ← x approaches 3

x	2.9	2.99	2.999	3.0	3.001	3.01	3.1
$f(x)$	−10	−100	−1000		1000	100	10

The pattern of outputs suggests that for values of x approaching 3 from the left, $f(x)$ becomes increasingly more negative. For values of x approaching 3 from the right, $f(x)$ becomes increasingly more positive. Therefore, $\lim_{x \to 3} f(x)$ does not exist.

3. Because $\lim_{x \to -3} f(x)$ exists, but $f(-3)$ is undefined, $f(x)$ has a removable discontinuity at $x = -3$. Because $f(x)$ decreases without bound as x approaches 3 from the left and increases without bound as x approaches 3 from the right, $f(x)$ has an infinite discontinuity at $x = 3$. The graph of $f(x)$ in Figure 1.3.3 supports these conclusions.

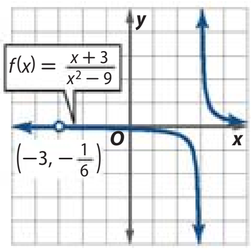


Figure 1.3.3

GuidedPractice

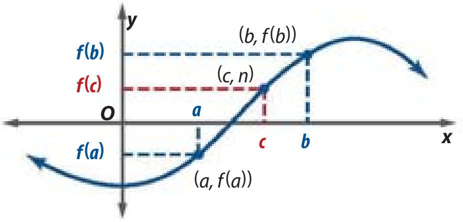
2A. $f(x) = \dfrac{1}{x^2}$; at $x = 0$

2B. $f(x) = \begin{cases} 5x + 4 & \text{if } x > 2 \\ 2 - x & \text{if } x \leq 2 \end{cases}$; at $x = 2$

If a function is continuous, you can approximate the location of its zeros by using the Intermediate Value Theorem and its corollary The Location Principle.

KeyConcept Intermediate Value Theorem

If $f(x)$ is a continuous function and $a < b$ and there is a value n such that n is between $f(a)$ and $f(b)$, then there is a number c, such that $a < c < b$ and $f(c) = n$.



Corollary: The Location Principle If $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c, such that $a < c < b$ and $f(c) = 0$. That is, there is a zero between a and b.

Example 3 Approximate Zeros

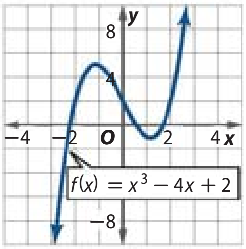
Determine between which consecutive integers the real zeros of each function are located on the given interval.

a. $f(x) = x^3 - 4x + 2$; $[-4, 4]$

x	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	−46	**−13**	**2**	5	**2**	**−1**	**2**	17	50

Because $f(-3)$ is negative and $f(-2)$ is positive, by the Location Principle, $f(x)$ has a zero between -3 and -2. The value of $f(x)$ also changes sign for $0 \le x \le 1$ and $1 \le x \le 2$. This indicates the existence of real zeros in each of these intervals.

The graph of $f(x)$ shown at the right supports the conclusion that there are real zeros between -3 and -2, 0 and 1, and 1 and 2.

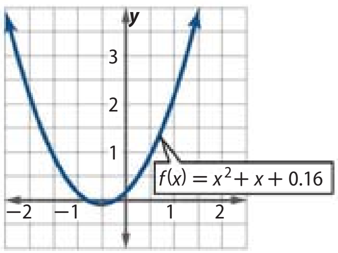


b. $f(x) = x^2 + x + 0.16$; $[-3, 3]$

x	−3	−2	−1	0	1	2	3
$f(x)$	6.16	2.16	0.16	0.16	2.16	6.16	12.16

The values of $f(x)$ do not change sign for the x-values used. However, as the x-values approach -1 from the left, $f(x)$ decreases, then begins increasing at $x = 0$. So, there may be real zeros between consecutive integers -1 and 0. Graph the function to verify.

The graph of $f(x)$ crosses the x-axis twice on the interval $[-1, 0]$, so there are real zeros between -1 and 0.



StudyTip

Approximating Zeros with No Sign Changes While a sign change on an interval *does* indicate the location of a real zero, the absence of a sign change *does not* indicate that there are no real zeros on that interval. The best method of checking this is to graph the function.

GuidedPractice

3A. $f(x) = \dfrac{x^2 - 6}{x + 4}$; $[-3, 4]$

3B. $f(x) = 8x^3 - 2x^2 - 5x - 1$; $[-5, 0]$

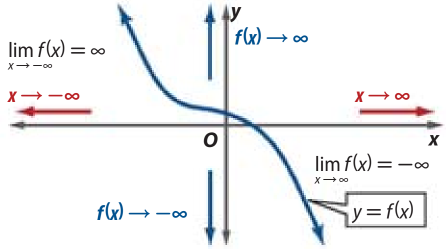
2 End Behavior

The **end behavior** of a function describes how a function *behaves* at either *end* of the graph. That is, end behavior is what happens to the value of $f(x)$ as x increases or decreases without bound—becoming greater and greater or more and more negative. To describe the end behavior of a graph, you can use the concept of a limit.

> **ReadingMath**
>
> **Limits** The expression $\lim\limits_{x \to \infty} f(x)$ is read *the limit of* $f(x)$ *as* x *approaches positive infinity.* The expression $\lim\limits_{x \to -\infty} f(x)$ is read *the limit of* $f(x)$ *as* x *approaches negative infinity.*

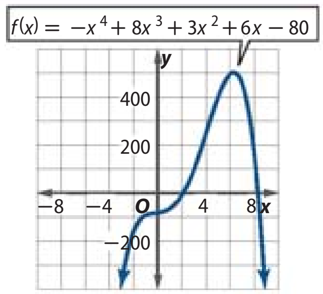
Left-End Behavior	Right-End Behavior
$\lim\limits_{x \to -\infty} f(x)$	$\lim\limits_{x \to \infty} f(x)$

One possibility for the end behavior of the graph of a function is for the value of $f(x)$ to increase or decrease without bound. This end behavior is described by saying that $f(x)$ approaches positive or negative infinity.



Example 4 Graphs that Approach Infinity

Use the graph of $f(x) = -x^4 + 8x^3 + 3x^2 + 6x - 80$ to describe its end behavior. Support the conjecture numerically.



Analyze Graphically

In the graph of $f(x)$, it appears that $\lim\limits_{x \to -\infty} f(x) = -\infty$ and $\lim\limits_{x \to \infty} f(x) = -\infty$.

Support Numerically

Construct a table of values to investigate function values as $|x|$ increases. That is, investigate the value of $f(x)$ as the value of x becomes greater and greater or more and more negative.

← x approaches $-\infty$ | x approaches ∞ →

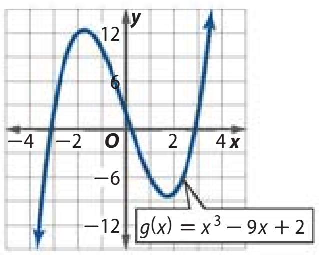
x	−10,000	−1000	−100	0	100	1000	10,000
$f(x)$	$-1 \cdot 10^{16}$	$-1 \cdot 10^{12}$	$-1 \cdot 10^{8}$	−80	$-1 \cdot 10^{8}$	$-1 \cdot 10^{12}$	$-1 \cdot 10^{16}$

The pattern of outputs suggests that as x approaches $-\infty$, $f(x)$ approaches $-\infty$ and as x approaches ∞, $f(x)$ approaches $-\infty$. This supports the conjecture.

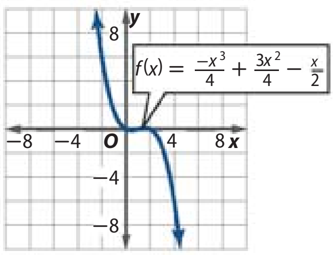
GuidedPractice

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

4A.



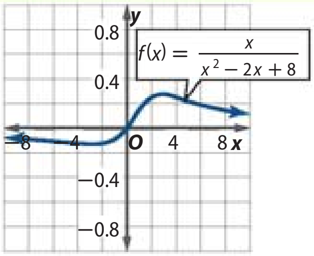
4B.



Instead of $f(x)$ being unbounded, approaching ∞ or $-\infty$ as $|x|$ increases, some functions approach, but never reach, a fixed value.

Example 5 Graphs that Approach a Specific Value

Use the graph of $f(x) = \dfrac{x}{x^2 - 2x + 8}$ to describe its end behavior. Support the conjecture numerically.



Analyze Graphically

In the graph of $f(x)$, it appears that $\lim\limits_{x \to -\infty} f(x) = 0$ and $\lim\limits_{x \to \infty} f(x) = 0$.

Support Numerically

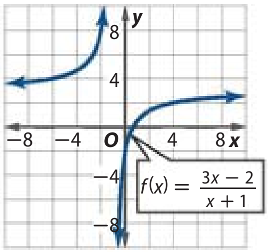
	x approaches $-\infty$				*x* approaches ∞		
x	−10,000	−1000	−100	0	100	1000	10,000
f(x)	$-1 \cdot 10^{-4}$	−0.001	−0.01	0	0.01	0.001	$1 \cdot 10^{-4}$

The pattern of outputs suggests that as x approaches $-\infty$, $f(x)$ approaches 0 and as x approaches ∞, $f(x)$ approaches 0. This supports the conjecture.

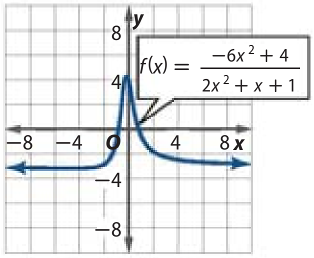
GuidedPractice

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

5A.



5B.

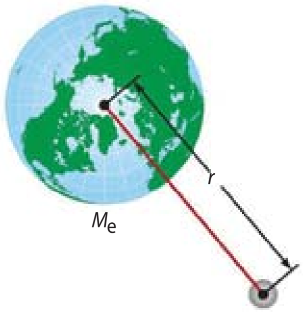


PhotoLink/Getty Images

Knowing the end behavior of a function can help you solve real-world problems.

Real-World Example 6 Apply End Behavior

PHYSICS **Gravitational potential energy of an object is given by $U(r) = -\dfrac{GmM_e}{r}$, where G is Newton's gravitational constant, m is the mass of the object, M_e is the mass of Earth, and r is the distance from the object to the center of Earth as shown. What happens to the gravitational potential energy of the object as it moves farther and farther from Earth?**



We are asked to describe the end behavior $U(r)$ for large values of r. That is, we are asked to find $\lim\limits_{r \to \infty} U(r)$. Because G, m, and M_e are constant values, the product GmM_e is also a constant value. For increasing values of r, the fraction $-\dfrac{GmM_e}{r}$ will approach 0, so $\lim\limits_{r \to \infty} U(r) = 0$. Therefore, as an object moves farther from Earth, its gravitational potential energy approaches 0.

Real-WorldLink

The form $U(r) = -\dfrac{GmM_e}{r}$ for gravitational potential energy is most useful for calculating the velocity required to escape Earth's gravity, 25,000 miles per hour.

Source: *The Mechanical Universe*

GuidedPractice

6. **PHYSICS** Dynamic pressure is the pressure generated by the velocity of the moving fluid and is given by $q(v) = \dfrac{\rho v^2}{2}$, where ρ is the density of the fluid and v is the velocity of the fluid. What would happen to the dynamic pressure of a fluid if the velocity were to continuously increase?

Exercises

= Step-by-Step Solutions begin on page R29.

Determine whether each function is continuous at the given *x*-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*. (Examples 1 and 2)

1. $f(x) = \sqrt{x^2 - 4}$; at $x = -5$

2. $f(x) = \sqrt{x + 5}$; at $x = 8$

3. $h(x) = \frac{x^2 - 36}{x + 6}$; at $x = -6$ and $x = 6$

4. $h(x) = \frac{x^2 - 25}{x + 5}$; at $x = -5$ and $x = 5$

5. $g(x) = \frac{x}{x - 1}$; at $x = 1$

6. $g(x) = \frac{2 - x}{2 + x}$; at $x = -2$ and $x = 2$

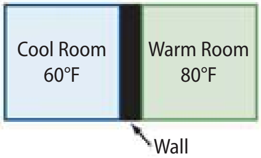
7. $h(x) = \frac{x - 4}{x^2 - 5x + 4}$; at $x = 1$ and $x = 4$

8. $h(x) = \frac{x(x - 6)}{x^3}$; at $x = 0$ and $x = 6$

9. $f(x) = \begin{cases} 4x - 1 & \text{if } x \leq -6 \\ -x + 2 & \text{if } x > -6 \end{cases}$; at $x = -6$

10. $f(x) = \begin{cases} x^2 - 1 & \text{if } x > -2 \\ x - 5 & \text{if } x \leq -2 \end{cases}$; at $x = -2$

11. **PHYSICS** A wall separates two rooms with different temperatures. The heat transfer in watts between the two rooms can be modeled by $f(w) = \frac{7.4}{w}$, where w is the wall thickness in meters. (Examples 1 and 2)



a. Determine whether the function is continuous at $w = 0.4$. Justify your answer using the continuity test.

b. Is the function continuous? Justify your answer using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

c. Graph the function to verify your conclusion from part **b**.

12. **CHEMISTRY** A solution must be diluted so it can be used in an experiment. Adding a 4-molar solution to a 10-molar solution will decrease the concentration. The concentration C of the mixture can be modeled by $C(x) = \frac{500 + 4x}{50 + x}$, where x is the number of liters of 4-molar solution added. (Examples 1 and 2)

a. Determine whether the function is continuous at $x = 10$. Justify the answer using the continuity test.

b. Is the function continuous? Justify your answer using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable* and describe what affect, if any, the discontinuity has on the concentration of the mixture.

c. Graph the function to verify your conclusion from part **b**.

Determine between which consecutive integers the real zeros of each function are located on the given interval. (Example 3)

13. $f(x) = x^3 - x^2 - 3$; $[-2, 4]$

14. $g(x) = -x^3 + 6x + 2$; $[-4, 4]$

15. $f(x) = 2x^4 - 3x^3 + x^2 - 3$; $[-3, 3]$

16. $h(x) = -x^4 + 4x^3 - 5x - 6$; $[3, 5]$

17. $f(x) = 3x^3 - 6x^2 - 2x + 2$; $[-2, 4]$

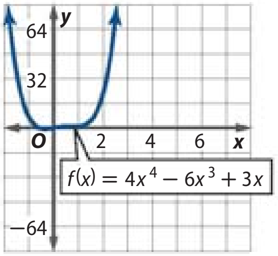
18. $g(x) = \frac{x^2 + 3x - 3}{x^2 + 1}$; $[-4, 3]$

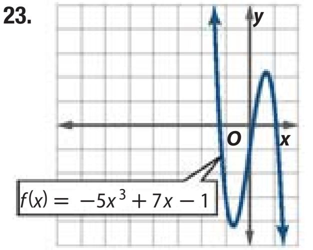
19. $h(x) = \frac{x^2 + 4}{x - 5}$; $[-2, 4]$

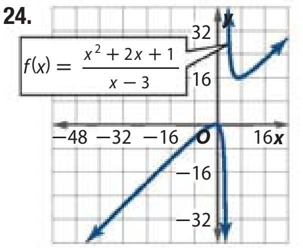
20. $f(x) = \sqrt{x^2 - 6} - 6$; $[3, 8]$

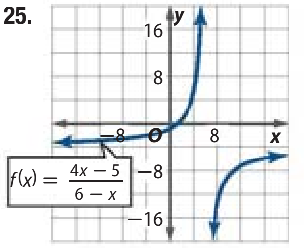
21. $g(x) = \sqrt{x^3 + 1} - 5$; $[0, 5]$

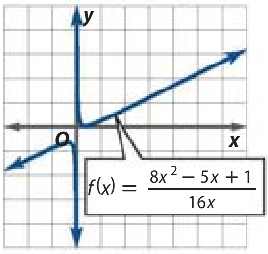
Use the graph of each function to describe its end behavior. Support the conjecture numerically. (Examples 4 and 5)

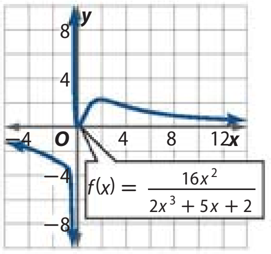
22. 

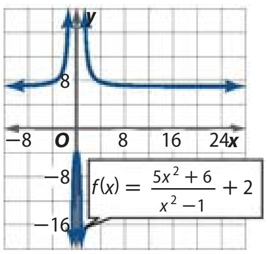
23. 

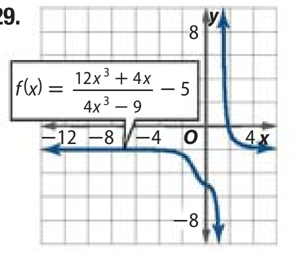
24. 

25. 

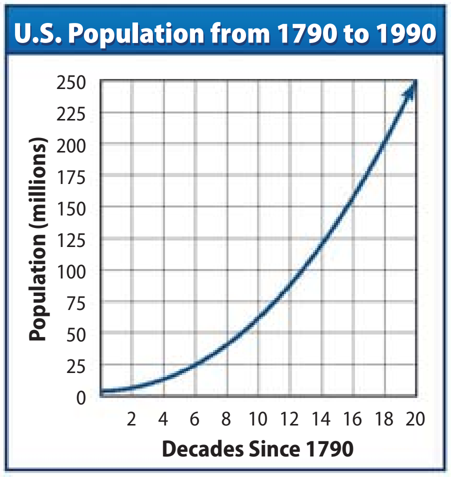
26. 

27. 

28. 

29. 

30. **POPULATION** The U.S. population from 1790 to 1990 can be modeled by $p(x) = 0.0057x^3 + 0.4895x^2 + 0.3236x + 3.8431$, where x is the number of decades after 1790. Use the end behavior of the graph to describe the population trend. Support the conjecture numerically. Does this trend seem realistic? Explain your reasoning. (Example 4)

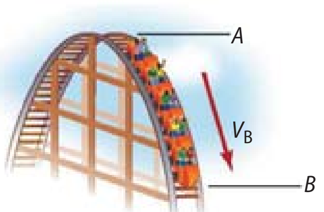


31. **CHEMISTRY** A catalyst is used to increase the rate of a chemical reaction. The reaction rate R, or the speed at which the reaction is occurring, is given by $R(x) = \frac{0.5x}{x + 12}$, where x is the concentration of the solution in milligrams of solute per liter of solution. (Example 5)

a. Graph the function using a graphing calculator.

b. What does the end behavior of the graph mean in the context of this experiment? Support the conjecture numerically.

32. **ROLLER COASTERS** The speed of a roller coaster after it drops from a height A to a height B is given by $f(h_A) = \sqrt{2g(h_A - h_B)}$, where h_A is the height at point A, h_B is the height at point B, and g is the acceleration due to gravity. What happens to $f(h_A)$ as h_B decreases to 0? (Example 6)



Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity. Explain your reasoning. (Example 6)

33. $q(x) = -\frac{24}{x}$

34. $f(x) = \frac{0.8}{x^2}$

35. $p(x) = \frac{x + 1}{x - 2}$

36. $m(x) = \frac{4 + x}{2x + 6}$

37. $c(x) = \frac{5x^2}{x^3 + 2x + 1}$

38. $k(x) = \frac{4x^2 - 3x - 1}{11x}$

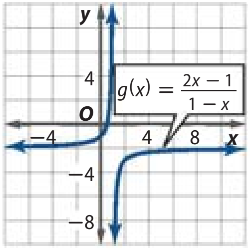
39. $h(x) = 2x^5 + 7x^3 + 5$

40. $g(x) = x^4 - 9x^2 + \frac{x}{4}$

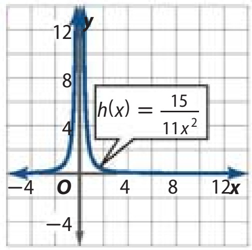
41. **PHYSICS** The kinetic energy of an object in motion can be expressed as $E(m) = \frac{p^2}{2m}$, where p is the momentum and m is the mass of the object. If sand is added to a moving railway car, what would happen as m continues to increase? (Example 6)

Use each graph to determine the x-value(s) at which each function is discontinuous. Identify the type of discontinuity. Then use the graph to describe its end behavior. Justify your answers.

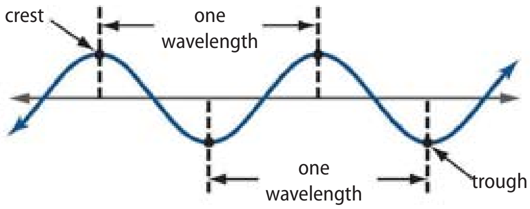
42.



43



44. **PHYSICS** The wavelength λ of a periodic wave is the distance between consecutive corresponding points on the wave, such as two crests or troughs.



The frequency f, or number of wave crests that pass any given point during a given period of time, is given by $f(\lambda) = \frac{c}{\lambda}$, where c is the speed of light or $2.99 \cdot 10^8$ meters per second.

a. Graph the function using a graphing calculator.

b. Use the graph to describe the end behavior of the function. Support your conjecture numerically.

c. Is the function continuous? If not, identify and describe any points of discontinuity.

GRAPHING CALCULATOR **Graph each function and determine whether it is continuous. If discontinuous, identify and describe any points of discontinuity. Then describe its end behavior and locate any zeros.**

45. $f(x) = \frac{x^2}{x^3 - 4x^2 + x + 6}$

46. $g(x) = \frac{x^2 - 9}{x^3 - 5x^2 - 18x + 72}$

47. $h(x) = \frac{4x^2 + 11x - 3}{x^2 + 3x - 18}$

48. $h(x) = \frac{x^3 - 4x^2 - 29x - 24}{x^2 - 2x - 15}$

49. $h(x) = \frac{x^3 - 5x^2 - 26x + 120}{x^2 + x - 12}$

50. **VEHICLES** The number A of alternative-fueled vehicles in use in the United States from 2000 to 2010 can be approximated by $f(t) = 5420t^2 - 14{,}726t + 531{,}750$, where t represents the number of years since 2000.

a. Graph the function.

b. About how many alternative-fueled vehicles were there in the United States in 2008?

c. As time goes by, what will the number of alternative-fueled vehicles approach, according to the model? Do you think that the model is valid after 2010? Explain.

GRAPHING CALCULATOR **Graph each function, and describe its end behavior. Support the conjecture numerically, and provide an effective viewing window for each graph.**

51. $f(x) = -x^4 + 12x^3 + 4x^2 - 4$

52. $g(x) = x^5 - 20x^4 + 2x^3 - 5$

53. $f(x) = \dfrac{16x^2}{x^2 + 15x}$

54. $g(x) = \dfrac{8x - 24x^3}{14 + 2x^3}$

55. **BUSINESS** Gabriel is starting a small business screen-printing and selling T-shirts. Each shirt costs \$3 to produce. He initially invested \$4000 for a screen printer and other business needs.

a. Write a function to represent the average cost per shirt as a function of the number of shirts sold n.

b. Use a graphing calculator to graph the function.

c. As the number of shirts sold increases, what value does the average cost approach?

56. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate limits. Consider $f(x) = \dfrac{ax^3 + b}{cx^3 + d}$, where a and c are nonzero integers, and b and d are integers.

a. **TABULAR** Let $c = 1$, and choose three different sets of values for a, b, and d. Write the function with each set of values. Copy and complete the table below.

$c = 1$				
a	b	d	$\lim_{x\to\infty} f(x)$	$\lim_{x\to-\infty} f(x)$

b. **TABULAR** Choose three different sets of values for each variable: one set with $a > c$, one set with $a < c$, and one set with $a = c$. Write each function, and create a table as you did in part **a**.

c. **ANALYTICAL** Make a conjecture about the limit of $f(x) = \dfrac{ax^3 + b}{cx^3 + d}$ as x approaches positive and negative infinity.

57. Graph several different functions of the form $f(x) = x^n + ax^{n-1} + bx^{n-2}$, where n, a, and b are integers, $n \geq 2$.

a. Make a conjecture about the end behavior of the function when n is positive and even. Include at least one graph to support your conjecture.

b. Make a conjecture about the end behavior of the function when n is positive and odd. Include at least one graph to support your conjecture.

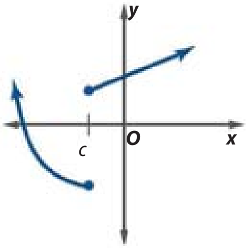
H.O.T. Problems Use Higher-Order Thinking Skills

REASONING **Determine whether each function has an *infinite*, *jump*, or *removable* discontinuity at $x = 0$. Explain.**

58. $f(x) = \dfrac{x^5 + x^6}{x^5}$

59. $f(x) = \dfrac{x^4}{x^5}$

60. **ERROR ANALYSIS** Keenan and George are determining whether the relation graphed below is continuous at point c. Keenan thinks that it is the graph of a function $f(x)$ that is discontinuous at point c because $\lim_{x\to c} f(x) = f(c)$ from only one side of c. George thinks that the graph is not a function because when $x = c$, the relation has two different y-values. Is either of them correct? Explain your reasoning.



61 **CHALLENGE** Determine the values of a and b so that f is continuous.

$$f(x) = \begin{cases} x^2 + a & \text{if } x \geq 3 \\ bx + a & \text{if } -3 < x < 3 \\ \sqrt{-b - x} & \text{if } x \leq -3 \end{cases}$$

REASONING **Find $\lim_{x\to-\infty} f(x)$ for each of the following. Explain your reasoning.**

62. $\lim_{x\to\infty} f(x) = -\infty$ and f is an even function.

63. $\lim_{x\to\infty} f(x) = -\infty$ and f is an odd function.

64. $\lim_{x\to\infty} f(x) = \infty$ and the graph of f is symmetric with respect to the origin.

65. $\lim_{x\to\infty} f(x) = \infty$ and the graph of f is symmetric with respect to the y-axis.

66. **WRITING IN MATH** Provide an example of a function with a removable discontinuity. Explain how this discontinuity can be eliminated. How does eliminating the discontinuity affect the function?

Spiral Review

GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.** (Lesson 1-2)

67. $h(x) = \sqrt{x^2 - 16}$

68. $f(x) = \dfrac{2x + 1}{x}$

69. $g(x) = x^5 - 5x^3 + x$

State the domain of each function. (Lesson 1-1)

70. $f(x) = \dfrac{4x + 6}{x^2 + 3x + 2}$

71. $g(x) = \dfrac{x + 3}{x^2 - 2x - 10}$

72. $g(a) = \sqrt{2 - a^2}$

73. **POSTAL SERVICE** The U.S. Postal Service uses five-digit ZIP codes to route letters and packages to their destinations. (Lesson 0-7)

a. How many ZIP codes are possible if the numbers 0 through 9 are used for each of the five digits?

b. Suppose that when the first digit is 0, the second, third, and fourth digits cannot be 0. How many five-digit ZIP codes are possible if the first digit is 0?

c. In 1983, the U.S. Postal Service introduced the ZIP + 4, which added four more digits to the existing five-digit ZIP codes. Using the numbers 0 through 9, how many additional ZIP codes were possible?

Given $A = \begin{bmatrix} -4 & 10 & -2 \\ 3 & -3 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 8 & -5 & 4 \\ 4 & 9 & -3 \end{bmatrix}$**, solve each equation for** X**.** (Lesson 0-6)

74. $3X - B = A$

75. $2B + X = 4A$

76. $A - 5X = B$

Solve each system of equations. (Lesson 0-5)

77.
$$\begin{aligned} 4x - 6y + 4z &= 12 \\ 6x - 9y + 6z &= 18 \\ 5x - 8y + 10z &= 20 \end{aligned}$$

78.
$$\begin{aligned} x + 2y + z &= 10 \\ 2x - y + 3z &= -5 \\ 2x - 3y - 5z &= 27 \end{aligned}$$

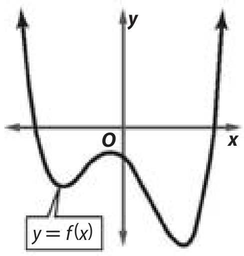
79.
$$\begin{aligned} 2x - y + 3z &= -2 \\ x + 4y - 2z &= 16 \\ 5x + y - z &= 14 \end{aligned}$$

Skills Review for Standardized Tests

80. **SAT/ACT** At Lincoln County High School, 36 students are taking either calculus or physics or both, and 10 students are taking both calculus and physics. If there are 31 students in the calculus class, how many students are there in the physics class?

A 5 **B** 8 **C** 11 **D** 15 **E** 21

81. Which of the following statements could be used to describe the end behavior of $f(x)$?



F $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

G $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

H $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$

J $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

82. **REVIEW** Amy's locker code includes three numbers between 1 and 45, inclusive. None of the numbers can repeat. How many possible locker permutations are there?

83. **REVIEW** Suppose a figure consists of three concentric circles with radii of 1 foot, 2 feet, and 3 feet. Find the probability that a point chosen at random lies in the outermost region (between the second and third circles).

A $\frac{1}{3}$ **B** $\frac{\pi}{9}$ **C** $\frac{4}{9}$ **D** $\frac{5}{9}$

LESSON 1-4 Extrema and Average Rates of Change

Then

- You found function values. (Lesson 1-1)

Now

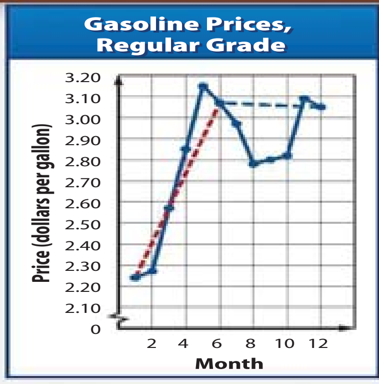
1. Determine intervals on which functions are increasing, constant, or decreasing, and determine maxima and minima of functions.
2. Determine the average rate of change of a function.

Why?

- The graph shows the average price of regular-grade gasoline in the U.S. from January to December.

The highest average price was about \$3.15 per gallon in May.

The slopes of the red and blue dashed lines show that the price of gasoline changed more rapidly in the first half of the year than in the second half.



NewVocabulary

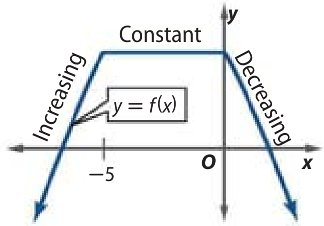
increasing
decreasing
constant
critical point
extrema
maximum
minimum
point of inflection
average rate of change
secant line

1 Increasing and Decreasing Behavior

An analysis of a function can also include a description of the intervals on which the function is increasing, decreasing, or constant.

Consider the graph of $f(x)$ shown. As you move from *left to right*, $f(x)$ is

- increasing or *rising* on $(-\infty, -5)$,
- constant or *flat* on $(-5, 0)$, and
- decreasing or *falling* on $(0, \infty)$.



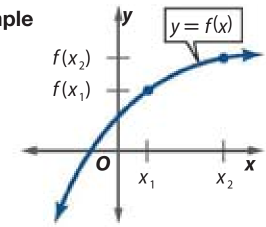
These graphical interpretations can also be described algebraically.

KeyConcept Increasing, Decreasing, and Constant Functions

Words A function f is **increasing** on an interval I if and only if for any two points in I, a positive change in x results in a positive change in $f(x)$.

Symbols For every x_1 and x_2 in an interval I, $f(x_1) < f(x_2)$ when $x_1 < x_2$.

Example

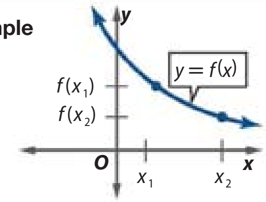


Interval: $(-\infty, \infty)$

Words A function f is **decreasing** on an interval I if and only if for any two points in I, a positive change in x results in a negative change in $f(x)$.

Symbols For every x_1 and x_2 in an interval I, $f(x_1) > f(x_2)$ when $x_1 < x_2$.

Example

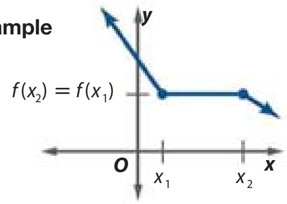


Interval: $(-\infty, \infty)$

Words A function f is **constant** on an interval I if and only if for any two points in I, a positive change in x results in a zero change in $f(x)$.

Symbols For every x_1 and x_2 in an interval I, $f(x_1) = f(x_2)$ when $x_1 < x_2$.

Example



Interval: (a, b)

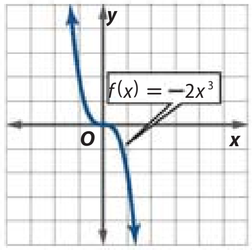
Example 1 Analyze Increasing and Decreasing Behavior

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.

a. $f(x) = -2x^3$

Analyze Graphically

When viewed from left to right, the graph of f falls for all real values of x. Therefore, we can conjecture that f is decreasing on $(-\infty, \infty)$.



Support Numerically

Create a table using values in the interval.

x	−8	−6	−4	−2	0	2	4	6	8
f(x)	1024	432	128	16	0	−16	−128	−432	−1024

The table shows that as x increases, $f(x)$ decreases. This supports the conjecture.

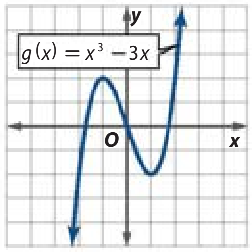
WatchOut!

Intervals A function is neither increasing nor decreasing at a point, so the symbols (and) should be used when describing the intervals on which a function is increasing or decreasing.

b. $g(x) = x^3 - 3x$

Analyze Graphically

From the graph, we can estimate that f is increasing on $(-\infty, -1)$, decreasing on $(-1, 1)$, and increasing on $(1, \infty)$.



Support Numerically

Create a table of values using x-values in each interval.

$(-\infty, -1)$:

x	−13	−11	−9	−7	−5	−3
f(x)	−2158	−1298	−702	−322	−110	−18

$(-1, 1)$:

x	−0.75	−0.5	0	0.5	0.75
f(x)	1.828	1.375	0	−1.375	−1.828

$(1, \infty)$:

x	3	5	7	9	11	13
f(x)	18	110	322	702	1298	2158

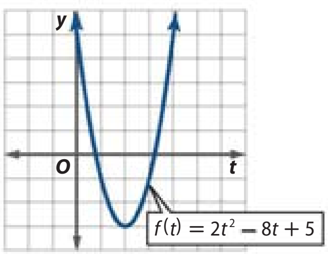
The tables show that as x increases to -1, $f(x)$ increases; as x increases from -1 to 1, $f(x)$ decreases; as x increases from 1, $f(x)$ increases. This supports the conjecture.

StudyTip

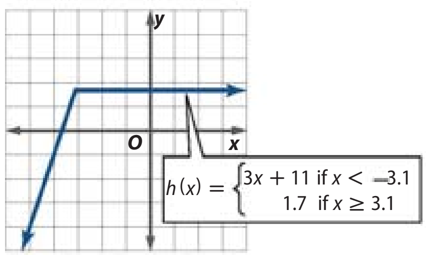
Increasing, Decreasing, and Constant Functions Functions that increase, decrease, or are constant for all x in their domain are called *increasing, decreasing,* or *constant functions,* respectively. The function in Example 1a is a decreasing function, while the function in Example 1b cannot be classified as increasing or decreasing because it has an interval where it is increasing and another interval where it is decreasing.

GuidedPractice

1A.



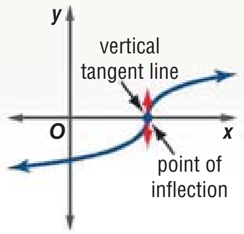
1B.



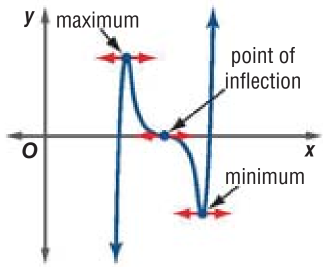
While a graphical approach to identify the intervals on which a function is increasing, decreasing, or constant can be supported numerically, calculus is often needed to confirm this behavior and to confirm that a function does not change its behavior beyond the domain shown.

StudyTip

Tangent Line Recall from geometry that a line is tangent to a curve if it intersects a curve in exactly one point.



Critical points of a function are those points at which a line drawn tangent to the curve is horizontal or vertical. **Extrema** are critical points at which a function changes its increasing or decreasing behavior. At these points, the function has a **maximum** or a **minimum** value, either relative or absolute. A **point of inflection** can also be a critical point. At these points, the graph changes its shape, but not its increasing or decreasing behavior. Instead, the curve changes from being bent upward to being bent downward, or vice versa.



KeyConcept Relative and Absolute Extrema

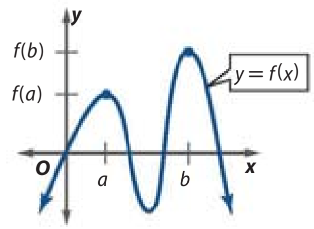
Words A *relative maximum* of a function f is the greatest value $f(x)$ can attain on some interval of the domain.

Symbols $f(a)$ is a relative maximum of f if there exists an interval (x_1, x_2) containing a such that $f(a) > f(x)$ for every $x \neq a$ in (x_1, x_2).

Words If a relative maximum is the greatest value a function f can attain over its entire domain, then it is the *absolute maximum.*

Symbols $f(b)$ is the absolute maximum of f if $f(b) > f(x)$ for every $x \neq b$, in the domain of f.

Model



$f(a)$ is a relative maximum of f.
$f(b)$ is the absolute maximum of f.

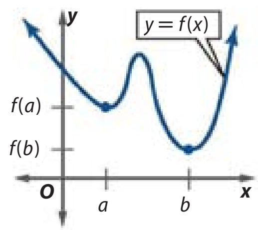
Words A *relative minimum* of a function f is the least value $f(x)$ can attain on some interval of the domain.

Symbols $f(a)$ is a relative minimum of f if there exists an interval (x_1, x_2) containing a such that $f(a) < f(x)$ for every $x \neq a$ in (x_1, x_2).

Words If a relative minimum is the least value a function f can attain over its entire domain, then it is the *absolute minimum.*

Symbols $f(b)$ is the absolute minimum of f if $f(b) < f(x)$ for every $x \neq b$, in the domain of f.

Model



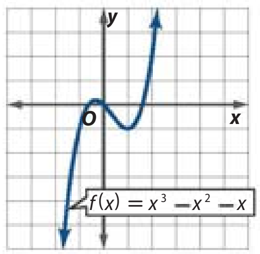
$f(a)$ is a relative minimum of f.
$f(b)$ is the absolute minimum of f.

ReadingMath

Plural Forms Using Latin, *maxima* is the plural form of *maximum*, *minima* is the plural form of *minimum*, and *extrema* is the plural form of *extremum*.

Example 2 Estimate and Identify Extrema of a Function

Estimate and classify the extrema for the graph of $f(x)$. Support the answers numerically.



Analyze Graphically

It appears that $f(x)$ has a relative maximum at $x = -0.5$ and a relative minimum at $x = 1$. It also appears that $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$, so we conjecture that this function has no absolute extrema.

Support Numerically

Choose x-values in half unit intervals on either side of the estimated x-value for each extremum, as well as one very large and one very small value for x.

x	−100	−1	**−0.5**	0	0.5	**1**	1.5	100
$f(x)$	$-1.0 \cdot 10^6$	−1.00	**0.125**	0	−0.63	**−1**	−0.38	$9.9 \cdot 10^5$

Because $f(-0.5) > f(-1)$ and $f(-0.5) > f(0)$, there is a relative maximum in the interval $(-1, 0)$ near -0.5. The approximate value of this relative maximum is $f(-0.5)$ or about 0.13.

StudyTip

Local Extrema Relative extrema are also called *local* extrema, and absolute extrema are also called *global* extrema.

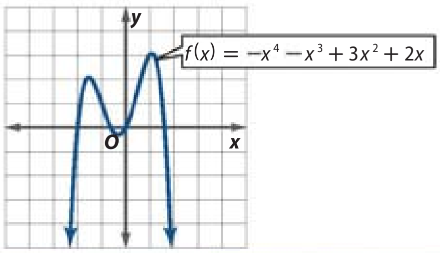
Likewise, because $f(1) < f(0.5)$ and $f(1) < f(1.5)$, there is a relative minimum in the interval $(0.5, 1.5)$ near 1. The approximate value of this relative maximum is $f(1)$ or -1.

$f(100) > f(-0.5)$ and $f(-100) < f(1)$, which supports our conjecture that f has no absolute extrema.

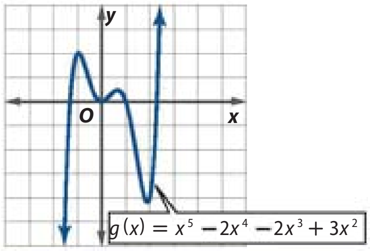
GuidedPractice

Estimate and classify the extrema for the graph of each function. Support the answers numerically.

2A.



2B.



Because calculus is needed to confirm the increasing and decreasing behavior of a function, calculus is also needed to confirm the relative and absolute extrema of a function. For now, however, you can use a graphing calculator to help you better approximate the location and function value of extrema.

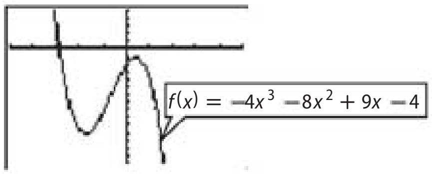
TechnologyTip

Zooming When locating maxima and minima, be sure to zoom in or out enough in order to see details and the overall appearance of the graph. The standard window may not tell the entire story.

Example 3 Use a Graphing Calculator to Approximate Extrema

GRAPHING CALCULATOR **Approximate to the nearest hundredth the relative or absolute extrema of $f(x) = -4x^3 - 8x^2 + 9x - 4$. State the x-value(s) where they occur.**

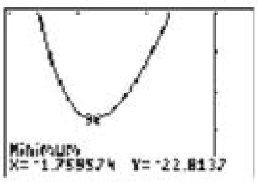
Graph the function and adjust the window as needed so that all of the graph's behavior is visible.



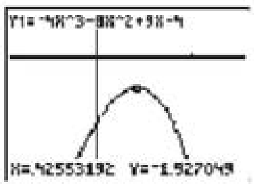
[−5, 5] scl: 1 by [−30, 10] scl: 4

From the graph of f, it appears that the function has one relative minimum in the interval $(-2, -1)$ and one relative maximum in the interval $(0, 1)$ of the domain. The end behavior of the graph suggests that this function has no absolute extrema.

Using the minimum and maximum options from the CALC menu of your graphing calculator, you can estimate that $f(x)$ has a relative minimum of -22.81 at $x \approx -1.76$ and a relative maximum of -1.93 at $x \approx 0.43$.



[−3, 0.5] scl: 1 by [−28, 12] scl: 4



[−0.9, 1.6] scl: 1 by [−7.3, 2.7] scl: 4

GuidedPractice

GRAPHING CALCULATOR **Approximate to the nearest hundredth the relative or absolute extrema of each function. State the x-value(s) where they occur.**

3A. $h(x) = 7 - 5x - 6x^2$

3B. $g(x) = 2x^3 - 4x^2 - x + 5$

Optimization is an application of mathematics where one searches for a maximum or a minimum quantity given a set of constraints. If a set of real-world quantities can be modeled by a function, the extrema of the function will indicate these optimal values.

Real-WorldLink

Florida produces 95% of the orange crop for orange juice in the United States. In a recent year, more than 880,000 tons of oranges were consumed in the United States.

Source: U.S. Department of Agriculture

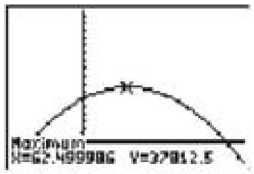
Real-World Example 4 Use Extrema for Optimization

AGRICULTURE Suppose each of the 75 orange trees in a Florida grove produces 400 oranges per season. Also suppose that for each additional tree planted in the orchard, the yield per tree decreases by 2 oranges. How many additional trees should be planted to achieve the greatest total yield?

Write a function $P(x)$ to describe the orchard yield as a function of x, the number of additional trees planted in the existing orchard.

orchard yield	=	number of trees in orchard	•	number of oranges produced per tree
$P(x)$	=	$(75 + x)$	•	$(400 - 2x)$

We want to maximize the orchard yield or $P(x)$. Graph this function using a graphing calculator. Then use the maximum option from the CALC menu to approximate the x-value that will produce the greatest value for $P(x)$.



[−100, 221.3] scl: 1 by [−12270.5, 87900] scl: 5000

The graph has a maximum of 37,812.5 for $x \approx 62.5$. So by planting an additional 62 trees, the orchard can produce a maximum yield of 37,812 oranges.

GuidedPractice

4. **CRAFTS** A glass candle holder is in the shape of a right circular cylinder that has a bottom and no top and has a total surface area of 10π square inches. Determine the radius and the height of the candle holder that will allow the maximum volume.

2 Average Rate of Change

In algebra, you learned that the slope between any two points on the graph of a linear function represents a *constant* rate of change. For a nonlinear function, the slope changes between different pairs of points, so we can only talk about the *average* rate of change between any two points.

KeyConcept Average Rate of Change

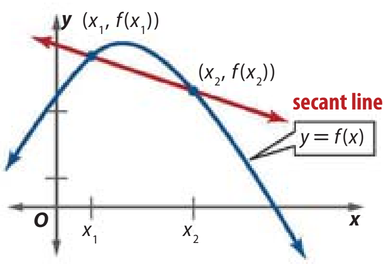
Words The **average rate of change** between any two points on the graph of f is the slope of the line through those points.

Geometry The line through two points on a curve is called a **secant line**. The slope of the secant line is denoted m_{sec}.

Symbols The average rate of change on the interval $[x_1, x_2]$ is

$$m_{sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Model



When the average rate of change over an interval is positive, the function increases on average over that interval. When the average rate of change is negative, the function decreases on average over that interval.

Example 5 Find Average Rates of Change

Find the average rate of change of $f(x) = -x^3 + 3x$ on each interval.

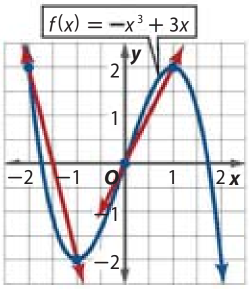


Figure 1.4.1

a. [−2, −1]

Use the Slope Formula to find the average rate of change of f on the interval [−2, −1].

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-2)}{-1 - (-2)} \quad \text{Substitute } -1 \text{ for } x_2 \text{ and } -2 \text{ for } x_1.$$

$$= \frac{[-(-1)^3 + 3(-1)] - [-(-2)^3 + 3(-2)]}{-1 - (-2)} \quad \text{Evaluate } f(-1) \text{ and } f(-2)$$

$$= \frac{-2 - 2}{-1 - (-2)} \text{ or } -4 \quad \text{Simplify.}$$

The average rate of change on the interval [−2, −1] is −4. Figure 1.4.1 supports this conclusion.

b. [0, 1]

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0} \quad \text{Substitute 1 for } x_2 \text{ and 0 for } x_1.$$

$$= \frac{2 - 0}{1 - 0} \text{ or } 2 \quad \text{Evaluate } f(1) \text{ and } f(0) \text{ and simplify.}$$

The average rate of change on the interval [0, 1] is 2. Figure 1.4.1 supports this conclusion.

GuidedPractice

Find the average rate of change of each function on the given interval.

5A. $f(x) = x^3 - 2x^2 - 3x + 2$; [2, 3]

5B. $f(x) = x^4 - 6x^2 + 4x$; [−5, −3]

Average rate of change has many real-world applications. One common application involves the average speed of an object traveling over a distance d or from a height h in a given period of time t. Because speed is distance traveled per unit time, the average speed of an object cannot be negative.

Real-WorldLink

Due to air resistance, a falling object will eventually reach a constant velocity known as *terminal velocity.* A skydiver with a closed parachute typically reaches terminal velocity of 120 to 150 miles per hour.

Source: *MSN Encarta*

Brand X/JupiterImages

Real-World Example 6 Find Average Speed

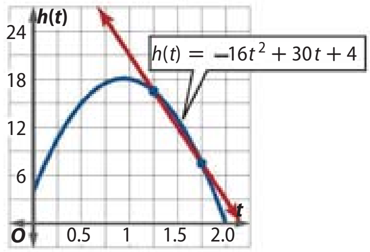
PHYSICS **The height of an object that is thrown straight up from a height of 4 feet above ground is given by $h(t) = -16t^2 + 30t + 4$, where t is the time in seconds after the object is thrown. Find and interpret the average speed of the object from 1.25 to 1.75 seconds.**

$$\frac{h(t_2) - h(t_1)}{t_2 - t_1} = \frac{h(1.75) - h(1.25)}{1.75 - 1.25} \quad \text{Substitute 1.75 for } t_2 \text{ and 1.25 for } t_1.$$

$$= \frac{[-16(1.75)^2 + 30(1.75) + 4] - [-16(1.25)^2 + 30(1.25) + 4]}{0.5} \quad \text{Evaluate } h(1.75) \text{ and } h(1.25).$$

$$= \frac{7.5 - 16.5}{0.5} \text{ or } -18 \quad \text{Simplify.}$$

The average rate of change on the interval is −18. Therefore, the average *speed* of the object from 1.25 to 1.75 seconds is 18 feet per second, and the distance the object is from the ground is decreasing on average over that interval, as shown in the figure at the right.



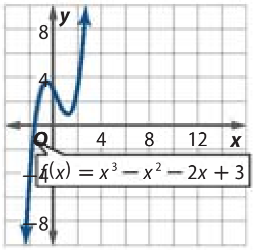
GuidedPractice

6. **PHYSICS** If wind resistance is ignored, the distance $d(t)$ in feet an object travels when dropped from a high place is given by $d(t) = 16t^2$, where t is the time is seconds after the object is dropped. Find and interpret the average speed of the object from 2 to 4 seconds.

Exercises

= Step-by-Step Solutions begin on page R29.

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically. (Example 1)

1\.


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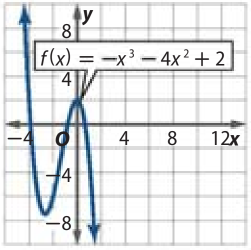

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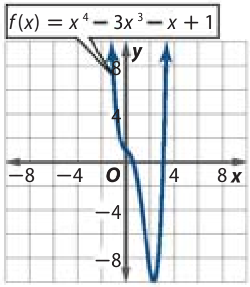

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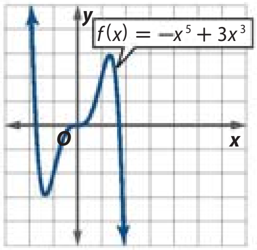

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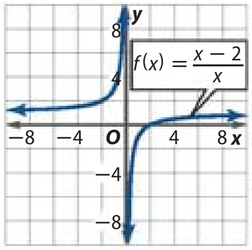

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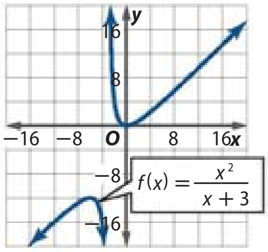

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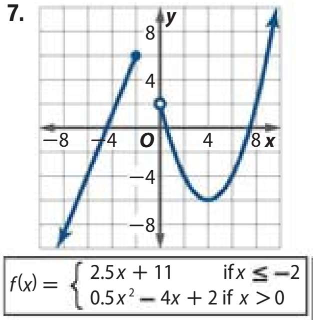

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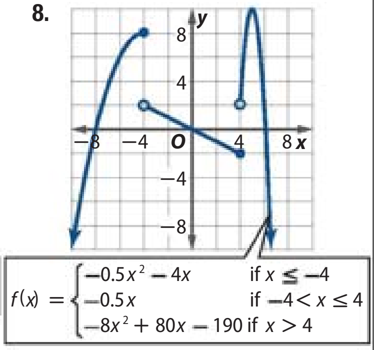

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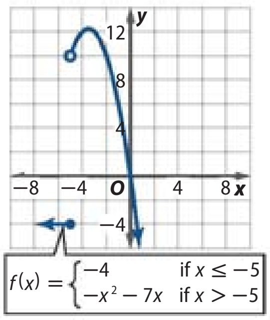
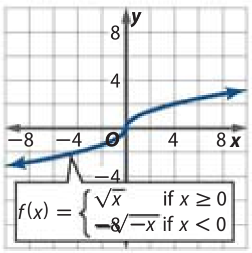


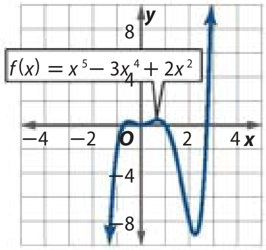
10\.


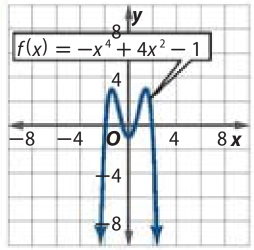
11\. **BASKETBALL** The height of a free-throw attempt can be modeled by $f(t) = -16t^2 + 23.8t + 5$, where t is time in seconds and $f(t)$ is the height in feet. (Example 2)

a. Graph the height of the ball.

b. Estimate the greatest height reached by the ball. Support the answer numerically.

Estimate and classify the extrema for the graph of each function. Support the answers numerically. (Example 2)

12\.


13\.


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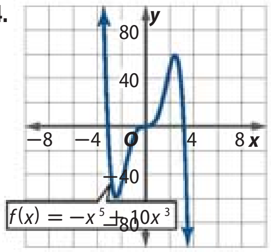
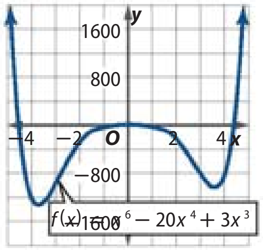


15\.


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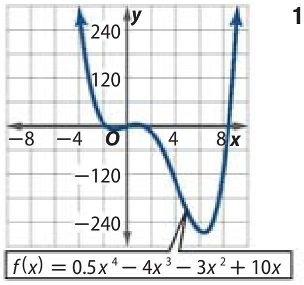

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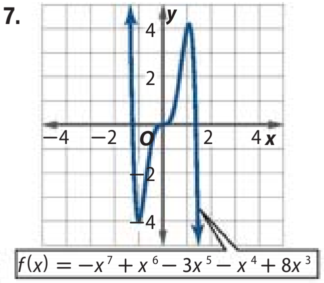

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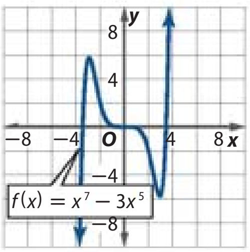

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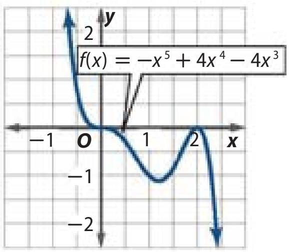

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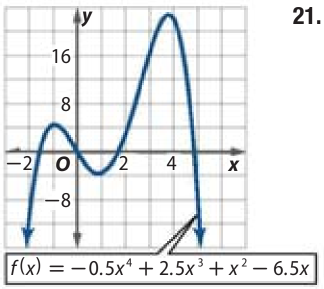

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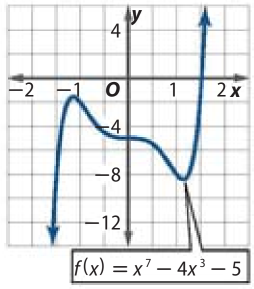


GRAPHING CALCULATOR **Approximate to the nearest hundredth the relative or absolute extrema of each function. State the x-value(s) where they occur.** (Example 3)

22. $f(x) = 3x^3 - 6x^2 + 8$

23. $g(x) = -2x^3 + 7x - 5$

24. $f(x) = -x^4 + 3x^3 - 2$

25. $f(x) = x^4 - 2x^2 + 5x$

26. $f(x) = x^5 - 2x^3 - 6x - 2$

27. $f(x) = -x^5 + 3x^2 + x - 1$

28. $g(x) = x^6 - 4x^4 + x$

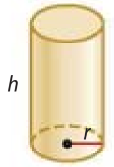
29. $g(x) = x^7 + 6x^2 - 4$

30. $f(x) = 0.008x^5 - 0.05x^4 - 0.2x^3 + 1.2x^2 - 0.7x$

31. $f(x) = 0.025x^5 - 0.1x^4 + 0.57x^3 + 1.2x^2 - 3.5x - 2$

32. **GRAPHIC DESIGN** A graphic designer wants to create a rectangular graphic that has a 2-inch margin on each side and a 4-inch margin on the top and the bottom. The design, including the margins, should have an area of 392 square inches. What overall dimensions will maximize the size of the design, excluding the margins? (*Hint*: If one side of the design is x, then the other side is 392 divided by x.) (Example 4)

33. **GEOMETRY** Determine the radius and height that will maximize the volume of the drinking glass shown. Round to the nearest hundredth of an inch, if necessary. (Example 4)



$SA = 20.5\pi$ in²

Find the average rate of change of each function on the given interval. (Example 5)

34. $g(x) = -4x^2 + 3x - 4$; $[-1, 3]$

35. $g(x) = 3x^2 - 8x + 2$; $[4, 8]$

36. $f(x) = 3x^3 - 2x^2 + 6$; $[2, 6]$

37. $f(x) = -2x^3 - 4x^2 + 2x - 8$; $[-2, 3]$

38. $f(x) = 3x^4 - 2x^2 + 6x - 1$; $[5, 9]$

39. $f(x) = -2x^4 - 5x^3 + 4x - 6$; $[-1, 5]$

40. $h(x) = -x^5 - 5x^2 + 6x - 9$; $[3, 6]$

41. $h(x) = x^5 + 2x^4 + 3x - 12$; $[-5, -1]$

42. $f(x) = \frac{x - 3}{x}$; $[5, 12]$

43. $f(x) = \frac{x + 5}{x - 4}$; $[-6, 2]$

44. $f(x) = \sqrt{x + 8}$; $[-4, 4]$

45. $f(x) = \sqrt{x - 6}$; $[8, 16]$

46. **WEATHER** The average high temperature by month in Pensacola, Florida, can be modeled by $f(x) = -0.9x^2 + 13x + 43$, where x is the month and $x = 1$ represents January. Find the average rate of change for each time interval, and explain what this rate represents. (Example 6)

a. April to May

b. July to November

47. **COFFEE** The world coffee consumption from 1990 to 2000 can be modeled by $f(x) = -0.004x^4 + 0.077x^3 - 0.38x^2 + 0.46x + 12$, where x is the year, $x = 0$ corresponds with 1990, and the consumption is measured in millions of pounds. Find the average rate of change for each time interval. (Example 6)

a. 1990 to 2000

b. 1995 to 2000

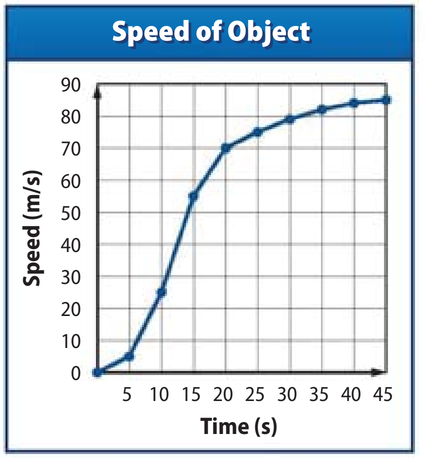
48. **TOURISM** Tourism in Hawaii for a given year can be modeled using $f(x) = 0.0635x^6 - 2.49x^5 + 37.67x^4 - 275.3x^3 + 986.6x^2 - 1547.1x + 1390.5$, where $1 \le x \le 12$, x represents the month, $x = 1$ corresponds with May 1st, and $f(x)$ represents the number of tourists in thousands.

a. Graph the equation.

b. During which month did the number of tourists reach its absolute maximum?

c. During which month did the number of tourists reach a relative maximum?

49. Use the graph to complete the following.



a. Find the average rate of change for [5, 15], [15, 20], and [25, 45].

b. Compare and contrast the nature of the speed of the object over these time intervals.

c. What conclusions can you make about the magnitude of the rate of change, the steepness of the graph, and the nature of the function?

50. **TECHNOLOGY** A computer company's research team determined that the profit per chip for a new processor chip can be modeled by $P(x) = -x^3 + 5x^2 + 8x$, where x is the sales price of the chip in hundreds of dollars.

a. Graph the function.

b. What is the optimum price per chip?

c. What is the profit per chip at the optimum price?

51. INCOME The average U.S. net personal income from 1997 to 2007 can be modeled by $I(x) = -1.465x^5 + 35.51x^4 - 277.99x^3 + 741.06x^2 + 847.8x + 25362$, $0 \le x \le 10$, where x is the number of years since 1997.

a. Graph the equation.

b. What was the average rate of change from 2000 to 2007? What does this value represent?

c. In what 4-year period was the average rate of change highest? lowest?

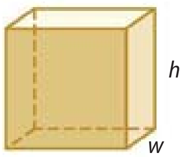
52. BUSINESS A company manufactures rectangular aquariums that have a capacity of 12 cubic feet. The glass used for the base of each aquarium is \$1 per square foot. The glass used for the sides is \$1.75 per square foot.

a. If the height and width of the aquarium are equal, find the dimensions that will minimize the cost to build an aquarium.

b. What is the minimum cost?

c. If the company also manufactures a cube-shaped aquarium with the same capacity, what is the difference in manufacturing costs?

53. PACKAGING Kali needs to design an enclosed box with a square base and a volume of 3024 cubic inches. What dimensions minimize the surface area of the box? Support your reasoning.



Sketch a graph of a function with each set of characteristics.

54. $f(x)$ is continuous and always increasing.

55. $f(x)$ is continuous and always decreasing.

56. $f(x)$ is continuous, always increasing, and $f(x) > 0$ for all values of x.

57. $f(x)$ is continuous, always decreasing, and $f(x) > 0$ for all values of x.

58. $f(x)$ is continuous, increasing for $x < -2$ and decreasing for $x > -2$.

59. $f(x)$ is continuous, decreasing for $x < 0$ and increasing for $x > 0$.

Determine the coordinates of the absolute extrema of each function. State whether each extremum is a *maximum* or *minimum* value.

60. $f(x) = 2(x - 3)^2 + 5$

61. $f(x) = -0.5(x + 5)^2 - 1$

62. $f(x) = -4|x - 22| + 65$

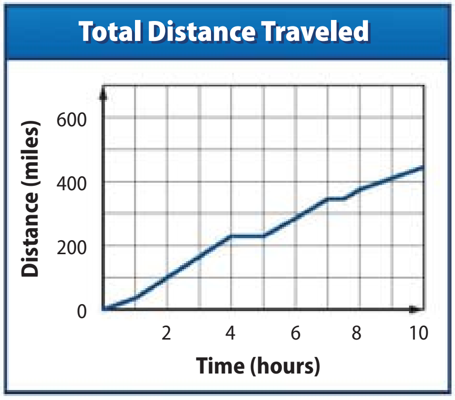
63. $f(x) = 4(3x - 7)^4 + 8$

64. $f(x) = (36 - x^2)^{0.5}$

65. $f(x) = -(25 - x^2)^{0.5}$

66. $f(x) = x^3 + x$

67. TRAVEL Each hour, Simeon recorded and graphed the total distance in miles his family drove during a trip. Give some reasons as to why the average rate of change varies and even appears constant during two intervals.



68. POINTS OF INFLECTION Determine which of the graphs in Exercises 1–10 and 12–21 have points of inflection that are critical points, and estimate the location of these points on each graph.

H.O.T. Problems Use Higher-Order Thinking Skills

OPEN ENDED Sketch a graph of a function with each set of characteristics.

69. infinite discontinuity at $x = -2$
increasing on $(-\infty, -2)$
increasing on $(-2, \infty)$
$f(-6) = -6$

70. continuous
average rate of change for [3, 8] is 4
decreasing on $(8, \infty)$
$f(-4) = 2$

71. REASONING What is the slope of the secant line from $(a, f(a))$ to $(b, f(b))$ when $f(x)$ is constant for the interval $[a, b]$? Explain your reasoning.

72. REASONING If the average rate of change of $f(x)$ on the interval (a, b) is positive, is $f(x)$ *sometimes*, *always*, or *never* increasing on (a, b)? Explain your reasoning.

73. CHALLENGE Use a calculator to graph $f(x) = \sin x$ in degree mode. Describe the relative extrema of the function and the window used for your graph.

74. REASONING A continuous function f has a relative minimum at c and is increasing as x increases from c. Describe the behavior of the function as x increases to c. Explain your reasoning.

75. WRITING IN MATH Describe how the average rate of change of a function relates to a function when it is increasing, decreasing, and constant on an interval.

Spiral Review

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*. (Lesson 1-3)

76. $f(x) = \sqrt{x^2 - 2};\ x = -3$

77. $f(x) = \sqrt{x + 1};\ x = 3$

78. $h(x) = \frac{x^2 - 25}{x + 5};\ x = -5$ and $x = 5$

GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.** (Lesson 1-2)

79. $f(x) = |x^5|$

80. $f(x) = \frac{x + 8}{x - 4}$

81. $g(x) = \frac{x^2}{x + 3}$

State the domain of each function. (Lesson 1-1)

82. $f(x) = \frac{3x}{x^2 - 5}$

83. $g(x) = \sqrt{x^2 - 9}$

84. $h(x) = \frac{x + 2}{\sqrt{x^2 - 7}}$

85. Find the values of x, y, and z for $3\begin{bmatrix} x & y - 1 \\ 4 & 3z \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ 6z & 3x + y \end{bmatrix}$. (Lesson 0-6)

86. If possible, find the solution of $y = x + 2z$, $z = -1 - 2x$, and $x = y - 14$. (Lesson 0-5)

Solve each equation. (Lesson 0-3)

87. $x^2 + 3x - 18 = 0$

88. $2a^2 + 11a - 21 = 0$

89. $z^2 - 4z - 21 = 0$

Simplify. (Lesson 0-2)

90. i^{19}

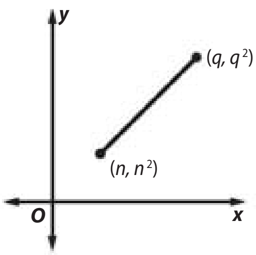
91. $(7 - 4i) + (2 - 3i)$

92. $\left(\frac{1}{2} + i\right) - (2 - i)$

93. **ELECTRICITY** The number of volts E produced by a circuit is given by $E = I \cdot Z$, where I is the current in amps and Z is the impedance in ohms. What number of amps is needed in a circuit that has an impedance of $3 - j$ ohms in order to produce $21 + 12j$ volts? (Lesson 0-2)

Skills Review for Standardized Tests

94. **SAT/ACT** In the figure, if $q \neq n$, what is the slope of the line segment?



A $q + n$

B $q - n$

C $\frac{q^2 + q}{n^2 - n}$

D $\frac{1}{q + n}$

E $\frac{1}{q - n}$

95. **REVIEW** When the number of a year is divisible by 4, then a leap year occurs. However, when the year is divisible by 100, then a leap year does not occur unless the year is divisible by 400. Which is an example of a leap year?

F 1882

G 1900

H 2000

J 2100

96. The function $f(x) = x^3 + 2x^2 - 4x - 6$ has a relative maximum and relative minimum located at which of the following x-values?

A relative maximum at $x \approx -0.7$, relative minimum at $x \approx 2$

B relative maximum at $x \approx -0.7$, relative minimum at $x \approx -2$

C relative maximum at $x \approx -2$, relative minimum at $x \approx 0.7$

D relative maximum at $x \approx 2$, relative minimum at $x \approx 0.7$

97. **REVIEW** A window is in the shape of an equilateral triangle. Each side of the triangle is 8 feet long. The window is divided in half by a support from one vertex to the midpoint of the side of the triangle opposite the vertex. Approximately how long is the support?

F 5.7 ft

G 6.9 ft

H 11.3 ft

J 13.9 ft

CHAPTER 1 Mid-Chapter Quiz

Lessons 1-1 through 1-4

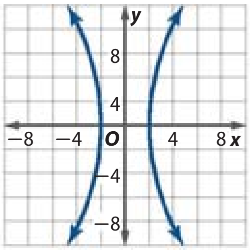
Determine whether each relation represents y as a function of x. (Lesson 1-1)

1. $3x + 7y = 21$

2.

x	-1	1	3	5	7
y	-1	3	7	11	15

3.



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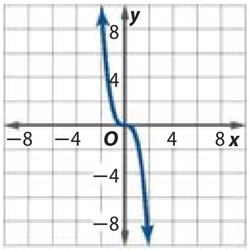
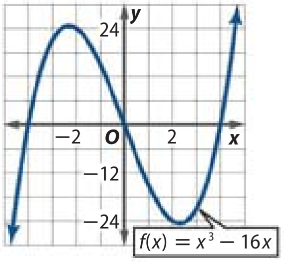


5. Evaluate $f(2)$ for $f(x) = \begin{cases} x^2 + 3x & \text{if } x < 2 \\ x + 10 & \text{if } x \geq 2 \end{cases}$. (Lesson 1-1)

6. **SPORTS** During a baseball game, a batter pops up the ball to the infield. After t seconds the height of the ball in feet can be modeled by $h(t) = -16t^2 + 50t + 5$. (Lesson 1-1)
   a. What is the baseball's height after 3 seconds?
   b. What is the relevant domain of this function? Explain your reasoning.

Use the graph of each function to find its y-intercept and zero(s). Then find these values algebraically. (Lesson 1-2)

7.



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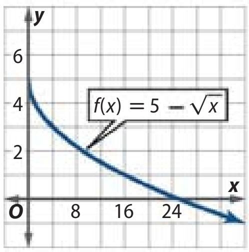
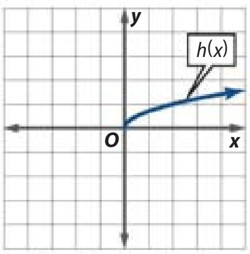


Use the graph of h to find the domain and range of each function. (Lesson 1-2)

9.



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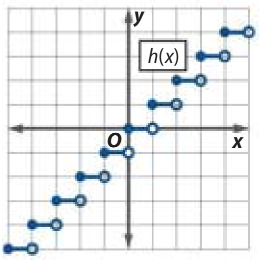


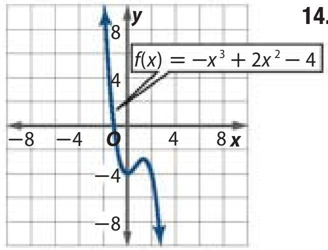
Determine whether each function is continuous at $x = 5$. Justify your answer using the continuity test. (Lesson 1-3)

11. $f(x) = \sqrt{x^2 - 36}$

12. $f(x) = \dfrac{x^2}{x + 5}$

Use the graph of each function to describe its end behavior. (Lesson 1-3)

13.



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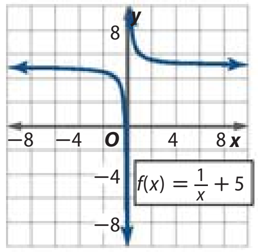
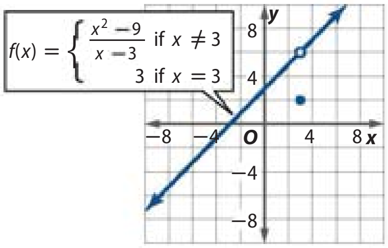


15. **MULTIPLE CHOICE** The graph of $f(x)$ contains a(n) ________ discontinuity at $x = 3$. (Lesson 1-3)



A undefined

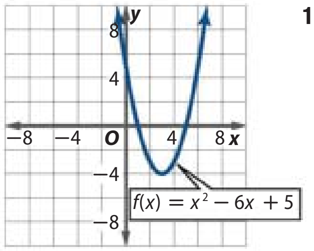
B infinite

C jump

D removable

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. (Lesson 1-4)

16.



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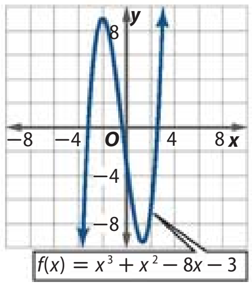


18. **PHYSICS** The height of an object dropped from 80 feet above the ground after t seconds is $f(t) = -16t^2 + 80$. What is the average speed for the object during the first 2 seconds after it is dropped? (Lesson 1-4)

LESSON 1-5

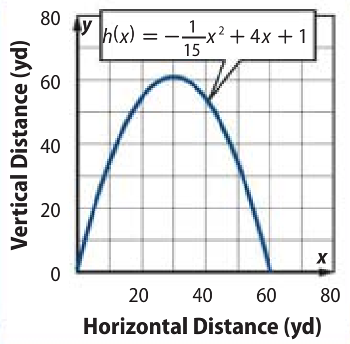
Parent Functions and Transformations

Then
- You analyzed graphs of functions. Lessons 1-2 through 1-4)

Now
- **1** Identify, graph, and describe parent functions.
- **2** Identify and graph transformations of parent functions.

Why?
- The path of a 60-yard punt can be modeled by the function at the right. This function is related to the basic quadratic function $f(x) = x^2$.

Punted Football



NewVocabulary
parent function
constant function
zero function
identity function
quadratic function
cubic function
square root function
reciprocal function
absolute value function
step function
greatest integer function
transformation
translation
reflection
dilation

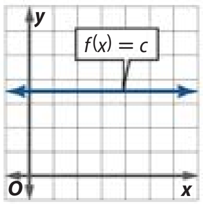
1 Parent Functions

A *family of functions* is a group of functions with graphs that display one or more similar characteristics. A **parent function** is the simplest of the functions in a family. This is the function that is transformed to create other members in a family of functions.

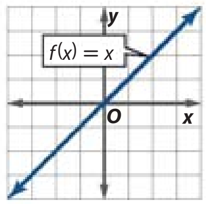
In this lesson, you will study eight of the most commonly used parent functions. You should already be familiar with the graphs of the following linear and polynomial parent functions.

KeyConcept Linear and Polynomial Parent Functions

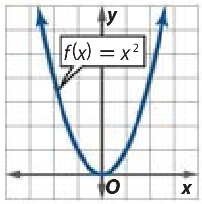
A **constant function** has the form $f(x) = c$, where c is any real number. Its graph is a horizontal line. When $c = 0$, $f(x)$ is the **zero function**.



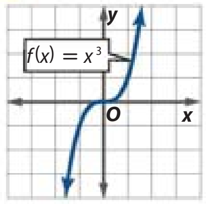
The **identity function** $f(x) = x$ passes through all points with coordinates (a, a).



The **quadratic function** $f(x) = x^2$ has a U-shaped graph.



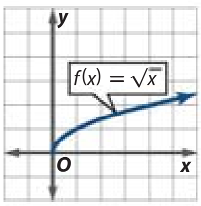
The **cubic function** $f(x) = x^3$ is symmetric about the origin.



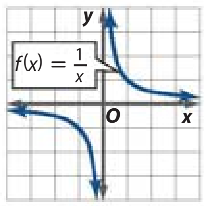
You should also be familiar with the graphs of both the square root and reciprocal functions.

KeyConcept Square Root and Reciprocal Parent Functions

The **square root function** has the form $f(x) = \sqrt{x}$.



The **reciprocal function** has the form $f(x) = \frac{1}{x}$.



Another parent function is the piecewise-defined absolute value function.

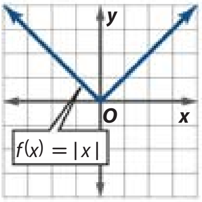
KeyConcept Absolute Value Parent Function

Words The **absolute value function**, denoted $f(x) = |x|$, is a V-shaped function defined as

$$f(x) = \begin{cases} -x & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}.$$

Examples $|-5| = 5$, $|0| = 0$, $|4| = 4$

Model



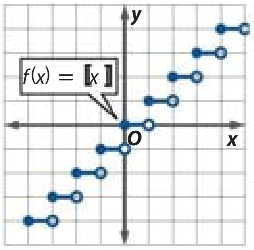
A piecewise-defined function in which the graph resembles a set of stairs is called a **step function**. The most well-known step function is the greatest integer function.

KeyConcept Greatest Integer Parent Function

Words The **greatest integer function**, denoted $f(x) = [\![x]\!]$, is defined as the greatest integer less than or equal to x.

Examples $[\![-4]\!] = -4$, $[\![-1.5]\!] = -2$, $\left[\!\!\left[\frac{1}{3}\right]\!\!\right] = 0$

Model



StudyTip

Floor Function The greatest integer function is also known as the *floor function*.

Using the tools you learned in Lessons 1-1 through 1-4, you can describe characteristics of each parent function. Knowing the characteristics of a parent function can help you analyze the shapes of more complicated graphs in that family.

Example 1 Describe Characteristics of a Parent Function

Describe the following characteristics of the graph of the parent function $f(x) = \sqrt{x}$: domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.

The graph of the square root function (Figure 1.5.1) has the following characteristics.

- The domain of the function is $[0, \infty)$, and the range is $[0, \infty)$.
- The graph has one intercept at $(0, 0)$.
- The graph has no symmetry. Therefore, $f(x)$ is neither odd nor even.
- The graph is continuous for all values in its domain.
- The graph begins at $x = 0$ and $\lim_{x \to \infty} f(x) = \infty$.
- The graph is increasing on the interval $(0, \infty)$.

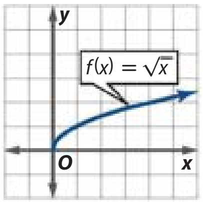


Figure 1.5.1

GuidedPractice

1. Describe the following characteristics of the graph of the parent function $f(x) = |x|$: domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.

2 Transformations

Transformations of a parent function can affect the appearance of the parent graph. *Rigid transformations* change only the position of the graph, leaving the size and shape unchanged. *Nonrigid transformations* distort the shape of the graph.

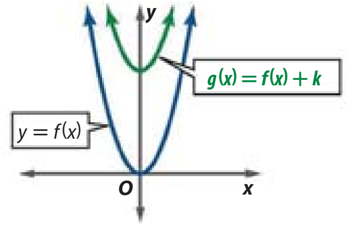
A **translation** is a rigid transformation that has the effect of shifting the graph of a function. A *vertical translation* of a function f shifts the graph of f up or down, while a *horizontal translation* shifts the graph left or right. Horizontal and vertical translations are examples of rigid transformations.

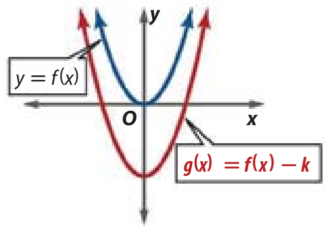
KeyConcept Vertical and Horizontal Translations

Vertical Translations

The graph of $g(x) = f(x) + k$ is the graph of $f(x)$ translated

- **k units up** when $k > 0$, and
- **k units down** when $k < 0$.

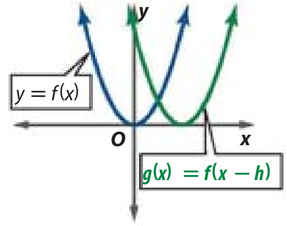


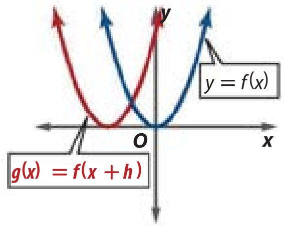


Horizontal Translations

The graph of $g(x) = f(x - h)$ is the graph of $f(x)$ translated

- **h units right** when $h > 0$, and
- **h units left** when $h < 0$.





Example 2 Graph Translations

Use the graph of $f(x) = |x|$ to graph each function.

a. $g(x) = |x| + 4$

This function is of the form $g(x) = f(x) + 4$. So, the graph of $g(x)$ is the graph of $f(x) = |x|$ translated 4 units up, as shown in Figure 1.5.2.

b. $g(x) = |x + 3|$

This function is of the form $g(x) = f(x + 3)$ or $g(x) = f[x - (-3)]$. So, the graph of $g(x)$ is the graph of $f(x) = |x|$ translated 3 units left, as shown in Figure 1.5.3.

c. $g(x) = |x - 2| - 1$

This function is of the form $g(x) = f(x - 2) - 1$. So, the graph of $g(x)$ is the graph of $f(x) = |x|$ translated 2 units right and 1 unit down, as shown in Figure 1.5.4.

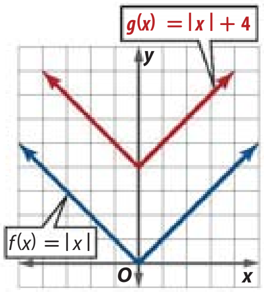


Figure 1.5.2

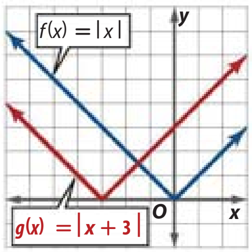


Figure 1.5.3

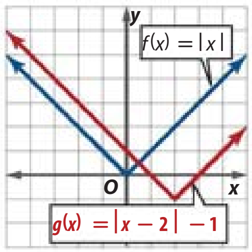


Figure 1.5.4

GuidedPractice **Use the graph of $f(x) = x^3$ to graph each function.**

2A. $h(x) = x^3 - 5$ **2B.** $h(x) = (x - 3)^3$ **2C.** $h(x) = (x + 2)^3 + 4$

TechnologyTip

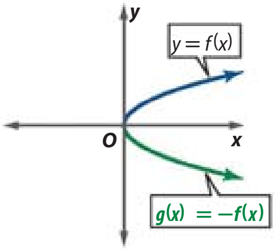
Translations You can translate a graph using a graphing calculator. Under [Y=], place an equation in Y1. Move to the Y2 line, and then press [VARS] [▶] [ENTER] [ENTER]. This will place Y1 in the Y2 line. Enter a number to translate the function. Press [GRAPH]. The two equations will be graphed in the same window.

Another type of rigid transformation is a **reflection**, which produces a mirror image of the graph of a function with respect to a specific line.

KeyConcept Reflections in the Coordinate Axes

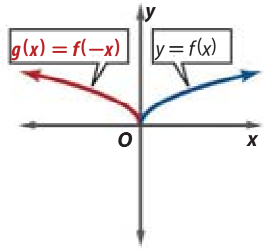
Reflection in x-axis

$g(x) = -f(x)$ is the graph of $f(x)$ **reflected in the x-axis**.

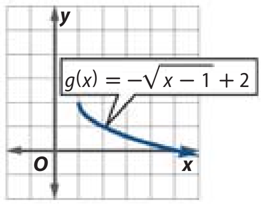


Reflection in y-axis

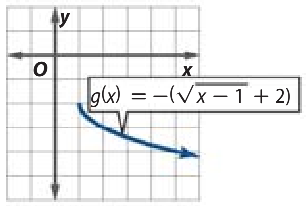
$g(x) = f(-x)$ is the graph of $f(x)$ **reflected in the y-axis**.



When writing an equation for a transformed function, be careful to indicate the transformations correctly. The graph of $g(x) = -\sqrt{x-1} + 2$ is different from the graph of $g(x) = -\left(\sqrt{x-1} + 2\right)$.



reflection of $f(x) = \sqrt{x}$ in the x-axis, then translated 1 unit to the right and 2 units up



translation of $f(x) = \sqrt{x}$ 1 unit to the right and 2 units up, then reflected in the x-axis

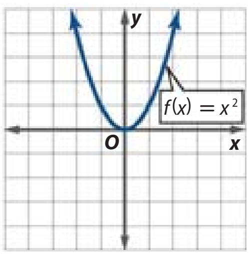
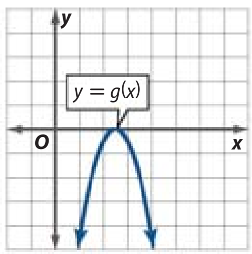


Figure 1.5.5

Example 3 Write Equations for Transformations

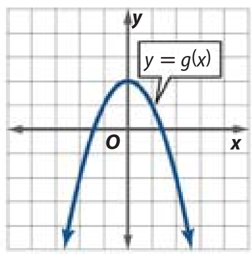
Describe how the graphs of $f(x) = x^2$ and $g(x)$ are related. Then write an equation for $g(x)$.

a.



The graph of $g(x)$ is the graph of $f(x) = x^2$ translated 2.5 units to the right and reflected in the x-axis. So, $g(x) = -(x - 2.5)^2$.

b.

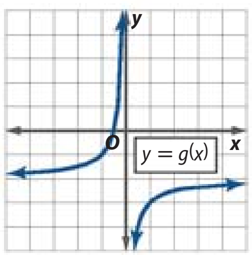


The graph of $g(x)$ is the graph of $f(x) = x^2$ reflected in the x-axis and translated 2 units up. So, $g(x) = -x^2 + 2$.

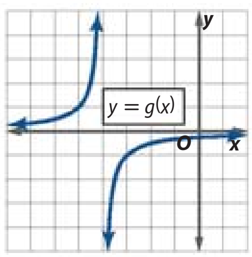
GuidedPractice

Describe how the graphs of $f(x) = \frac{1}{x}$ and $g(x)$ are related. Then write an equation for $g(x)$.

3A.



3B.



A **dilation** is a nonrigid transformation that has the effect of compressing (shrinking) or expanding (enlarging) the graph of a function vertically or horizontally.

StudyTip

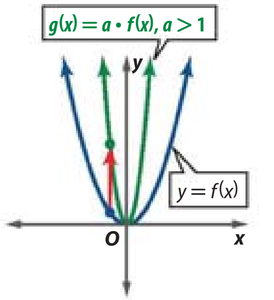
Dilations Sometimes pairs of dilations look similar such as a vertical expansion and a horizontal compression. It is not possible to tell which dilation a transformation is from the graph. You must compare the equation of the transformed function to the parent function.

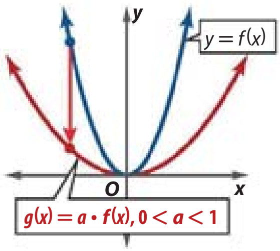
KeyConcept Vertical and Horizontal Dilations

Vertical Dilations

If a is a positive real number, then $g(x) = a \cdot f(x)$, is

- the graph of $f(x)$ **expanded vertically**, if $a > 1$.
- the graph of $f(x)$ **compressed vertically**, if $0 < a < 1$.

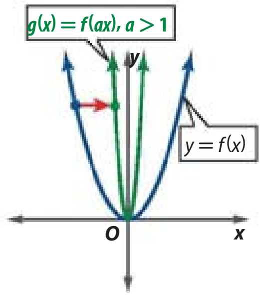


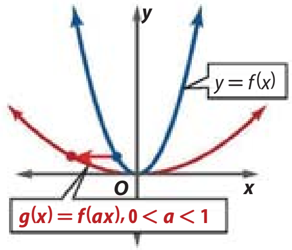


Horizontal Dilations

If a is a positive real number, then $g(x) = f(ax)$, is

- the graph of $f(x)$ **compressed horizontally**, if $a > 1$.
- the graph of $f(x)$ **expanded horizontally**, if $0 < a < 1$.



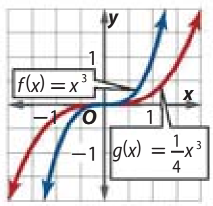


Example 4 Describe and Graph Transformations

Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

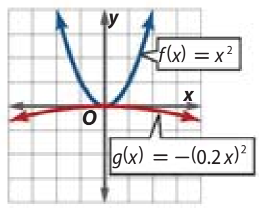
a. $g(x) = \frac{1}{4}x^3$

The graph of $g(x)$ is the graph of $f(x) = x^3$ compressed vertically because $g(x) = \frac{1}{4}x^3 = \frac{1}{4}f(x)$ and $0 < \frac{1}{4} < 1$.



b. $g(x) = -(0.2x)^2$

The graph of $g(x)$ is the graph of $f(x) = x^2$ expanded horizontally and then reflected in the x-axis because $g(x) = -(0.2x)^2 = -f(0.2x)$ and $0 < 0.2 < 1$.



GuidedPractice

4A. $g(x) = [\![x]\!] - 4$

4B. $g(x) = \frac{15}{x} + 3$

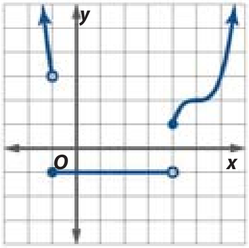
You can use what you have learned about transformations of functions to graph a piecewise-defined function.

Example 5 Graph a Piecewise-Defined Function

Graph $f(x) = \begin{cases} 3x^2 & \text{if } x < -1 \\ -1 & \text{if } -1 \le x < 4. \\ (x-5)^3 + 2 & \text{if } x \ge 4 \end{cases}$

On the interval $(-\infty, -1)$, graph $y = 3x^2$.
On the interval $[-1, 4)$, graph the constant function $y = -1$.
On the interval $[4, \infty)$, graph $y = (x-5)^3 + 2$.

Draw circles at $(-1, 3)$ and $(4, -1)$ and dots at $(-1, -1)$ and $(4, 1)$ because $f(-1) = -1$ and $f(4) = 1$.



GuidedPractice

Graph each function.

5A. $g(x) = \begin{cases} x - 5 & \text{if } x \le 0 \\ x^3 & \text{if } 0 < x \le 2 \\ \frac{2}{x} & \text{if } x > 2 \end{cases}$

5B. $h(x) = \begin{cases} (x+6)^2 & \text{if } x < -5 \\ 7 & \text{if } -5 \le x \le 2 \\ |4 - x| & \text{if } x > 2 \end{cases}$

You can also use what you have learned about transformations to transform functions that model real-world data or phenomena.

Real-WorldLink

The record for the longest punt in NFL history is 98 yards, kicked by Steve O'Neal on September 21, 1969.

Source: *National Football League*

Real-World Example 6 Transformations of Functions

FOOTBALL The path of a 60-yard punt can be modeled by $g(x) = -\frac{1}{15}x^2 + 4x + 1$**, where** $g(x)$ **is the vertical distance in yards of the football from the ground and** x **is the horizontal distance in yards such that** $x = 0$ **corresponds to the kicking team's 20-yard line.**

a. Describe the transformations of the parent function $f(x) = x^2$ **used to graph** $g(x)$**.**

Rewrite the function so that it is in the form $g(x) = a(x - h)^2 + k$ by completing the square.

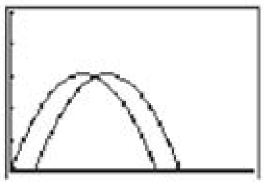
$g(x) = -\frac{1}{15}x^2 + 4x + 1$	Original function
$= -\frac{1}{15}(x^2 - 60x) + 1$	Factor $-\frac{1}{15}x^2 + 4x$.
$= -\frac{1}{15}(x^2 - 60x + 900) + 1 + \frac{1}{15}(900)$	Complete the square.
$= -\frac{1}{15}(x - 30)^2 + 61$	Write $x^2 - 60x + 900$ as a perfect square and simplify.

So, $g(x)$ is the graph of $f(x)$ translated 30 units right, compressed vertically, reflected in the x-axis, and then translated 61 units up.

b. Suppose the punt was from the kicking team's 30-yard line. Rewrite $g(x)$ **to reflect this change. Graph both functions on the same graphing calculator screen.**

A change of position from the kicking team's 20- to 30-yard line is a horizontal translation of 10 yards to the right, so subtract an additional 10 yards from inside the squared expression.

$g(x) = -\frac{1}{15}(x - 30 - 10)^2 + 61$ or $g(x) = -\frac{1}{15}(x - 40)^2 + 61$



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GuidedPractice

6. ELECTRICITY The current in amps flowing through a DVD player is described by $I(x) = \sqrt{\frac{x}{11}}$, where x is the power in watts and 11 is the resistance in ohms.

A. Describe the transformations of the parent function $f(x) = \sqrt{x}$ used to graph $I(x)$.

B. The resistance of a lamp is 15 ohms. Write a function to describe the current flowing through the lamp.

C. Graph the resistance for the DVD player and the lamp on the same graphing calculator screen.

Another nonridgid transformation involves absolute value.

TechnologyTip

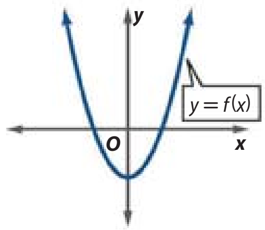
Absolute Value Transformations

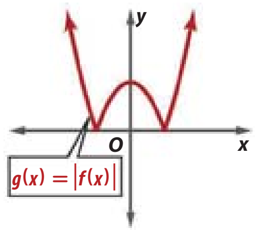
You can check your graph of an absolute value transformation by using your graphing calculator. You can also graph both functions on the same coordinate axes.

KeyConcept Transformations with Absolute Value

$g(x) = |f(x)|$

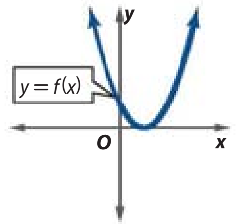
This transformation reflects any portion of the graph of $f(x)$ that is below the x-axis so that it is above the x-axis.

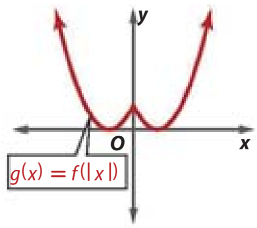




$g(x) = f(|x|)$

This transformation results in the portion of the graph of $f(x)$ that is to the left of the y-axis being replaced by a reflection of the portion to the right of the y-axis.





Example 7 Describe and Graph Transformations

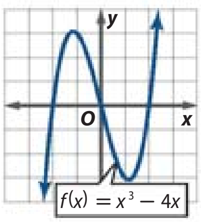
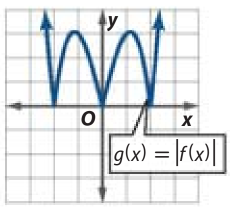


Figure 1.5.6

Use the graph of $f(x) = x^3 - 4x$ in Figure 1.5.6 to graph each function.

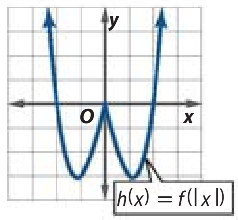
a. $g(x) = |f(x)|$

The graph of $f(x)$ is below the x-axis on the intervals $(-\infty, -2)$ and $(0, 2)$, so reflect those portions of the graph in the x-axis and leave the rest unchanged.



b. $h(x) = f(|x|)$

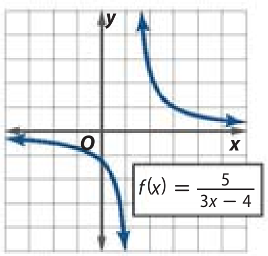
Replace the graph of $f(x)$ to the left of the y-axis with a reflection of the graph to the right of the y-axis.



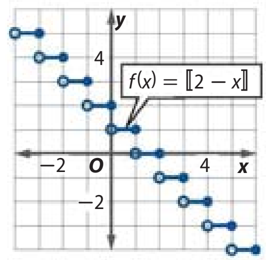
GuidedPractice

Use the graph of $f(x)$ shown to graph $g(x) = |f(x)|$ and $h(x) = f(|x|)$.

7A.



7B.



Exercises

● = Step-by-Step Solutions begin on page R29.

Describe the following characteristics of the graph of each parent function: domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing. (Example 1)

1. $f(x) = [\![x]\!]$
2. $f(x) = \frac{1}{x}$
3. $f(x) = x^3$
4. $f(x) = x^4$
5. $f(x) = c$
6. $f(x) = x$

Use the graph of $f(x) = \sqrt{x}$ to graph each function. (Example 2)

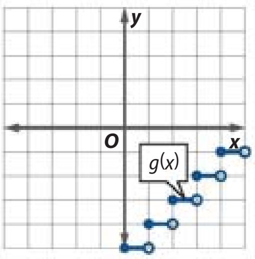
7. $g(x) = \sqrt{x - 4}$
8. $g(x) = \sqrt{x + 3}$
9. $g(x) = \sqrt{x + 6} - 4$
10. $g(x) = \sqrt{x - 7} + 3$

Use the graph of $f(x) = \frac{1}{x}$ to graph each function. (Example 2)

11. $g(x) = \frac{1}{x} + 4$
12. $g(x) = \frac{1}{x} - 6$
13. $g(x) = \frac{1}{x - 6} + 8$
14. $g(x) = \frac{1}{x + 7} - 4$

Describe how the graphs of $f(x) = [\![x]\!]$ and $g(x)$ are related. Then write an equation for $g(x)$. (Example 3)

15.



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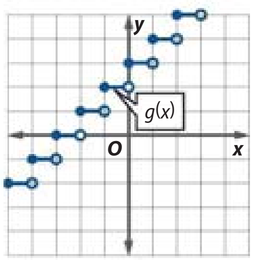

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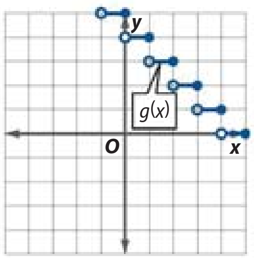

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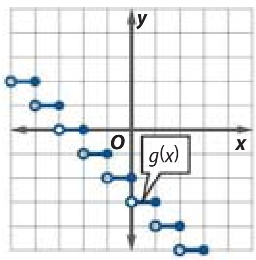
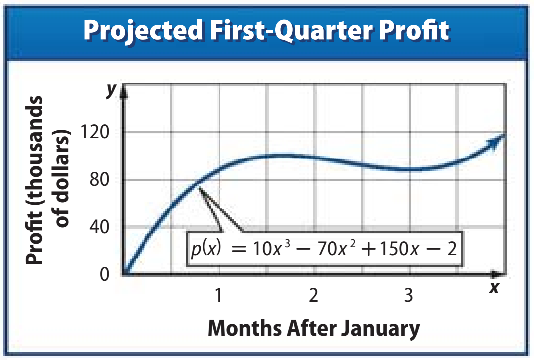
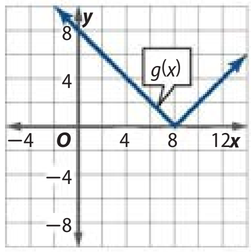


19. **PROFIT** An automobile company experienced an unexpected two-month delay on manufacturing of a new car. The projected profit of the car sales before the delay $p(x)$ is shown below. Describe how the graph of $p(x)$ and the graph of a projection including the delay $d(x)$ are related. Then write an equation for $d(x)$. (Example 3)



Describe how the graphs of $f(x) = |x|$ and $g(x)$ are related. Then write an equation for $g(x)$. (Example 3)

20.



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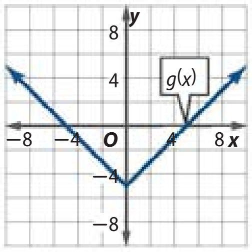

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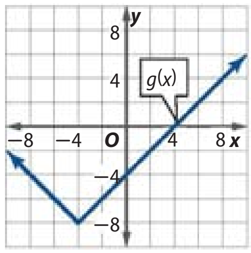

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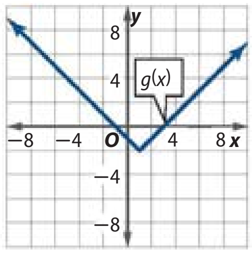


Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes. (Example 4)

24. $g(x) = 3|x| - 4$
25. $g(x) = 3\sqrt{x + 8}$
26. $g(x) = \frac{4}{x + 1}$
27. $g(x) = 2[\![x - 6]\!]$
28. $g(x) = -5[\![x - 2]\!]$
29. $g(x) = -2|x + 5|$
30. $g(x) = \frac{1}{6x} + 7$
31. $g(x) = \frac{\sqrt{x + 3}}{4}$

Graph each function. (Example 5)

32. $f(x) = \begin{cases} -x^2 & \text{if } x < -2 \\ 3 & \text{if } -2 \le x < 7 \\ (x - 5)^2 + 2 & \text{if } x \ge 7 \end{cases}$

33. $g(x) = \begin{cases} x + 4 & \text{if } x < -6 \\ \frac{1}{x} & \text{if } -6 \le x < 4 \\ 6 & \text{if } x \ge 4 \end{cases}$

34. $f(x) = \begin{cases} 4 & \text{if } x < -5 \\ x^3 & \text{if } -2 \le x \le 2 \\ \sqrt{x + 3} & \text{if } x > 3 \end{cases}$

35. $h(x) = \begin{cases} |x - 5| & \text{if } x < -3 \\ 4x - 3 & \text{if } -1 \le x < 3 \\ \sqrt{x} & \text{if } x \ge 4 \end{cases}$

36. $g(x) = \begin{cases} 2 & \text{if } x < -4 \\ x^4 - 3x^3 + 5 & \text{if } -1 \le x < 1 \\ [\![x]\!] + 1 & \text{if } x \ge 3 \end{cases}$

37. $f(x) = \begin{cases} -3x - 1 & \text{if } x \le -1 \\ 0.5x + 5 & \text{if } -1 < x \le 3 \\ -|x - 5| + 3 & \text{if } x > 3 \end{cases}$

38. **POSTAGE** The cost of a first-class postage stamp in the U.S. from 1988 to 2008 is shown in the table below. Use the data to graph a step function. (Example 5)

Year	Price (¢)
1988	25
1991	29
1995	32
1999	33
2001	34
2002	37
2006	39
2007	41
2008	42

39. **BUSINESS** A no-contract cell phone company charges a flat rate for daily access and \$0.10 for each minute. The cost of the plan can be modeled by $c(x) = 0.1[\![x]\!] + 1.99$, where x is the number of minutes used. (Example 6)

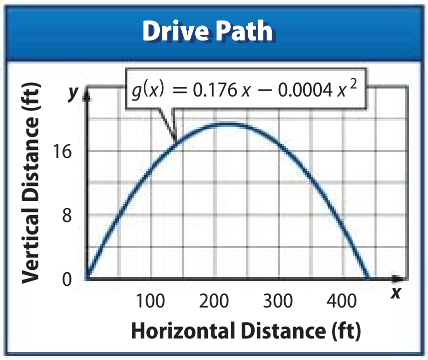
a. Describe the transformation(s) of the parent function $f(x) = [\![x]\!]$ used to graph $c(x)$.

b. The company offers another plan in which the daily access rate is \$2.49, and the per-minute rate is \$0.05. What function $d(x)$ can be used to describe the second plan?

c. Graph both functions on the same graphing calculator screen.

d. Would the cost of the plans ever equal each other? If so, at how many minutes?

40. **GOLF** The path of a drive can be modeled by the function shown, where $g(x)$ is the vertical distance in feet of the ball from the ground and x is the horizontal distance in feet such that $x = 0$ corresponds to the initial point. (Example 6)



a. Describe the transformation(s) of the parent function $f(x) = x^2$ used to graph $g(x)$.

b. If a second golfer hits a similar shot 30 feet farther down the fairway from the first player, what function $h(x)$ can be used to describe the second golfer's shot?

c. Graph both golfers' shots on the same graphing calculator screen.

d. At what horizontal and vertical distances do the paths of the two shots cross each other?

Use the graph of $f(x)$ to graph $g(x) = |f(x)|$ and $h(x) = f(|x|)$. (Example 7)

41. $f(x) = \frac{2}{x}$

42. $f(x) = \sqrt{x - 4}$

43. $f(x) = x^4 - x^3 - 4x^2$

44. $f(x) = \frac{1}{2}x^3 + 2x^2 - 8x - 2$

45. $f(x) = \frac{1}{x - 3} + 5$

46. $f(x) = \sqrt{x + 2} - 6$

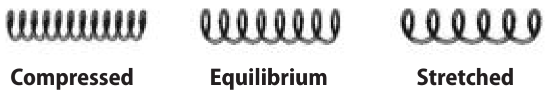
47. **TRANSPORTATION** In New York City, the standard cost for taxi fare is shown. One unit is equal to a distance of 0.2 mile or a time of 60 seconds when the car is not in motion.

a. Write a greatest integer function $f(x)$ that would represent the cost for units of cab fare, where $x > 0$. Round to the nearest unit.

b. Graph the function.

c. How would the graph of $f(x)$ change if the fare for the first unit increased to \$3.70 while the cost per unit remained at \$0.40? Graph the new function.

48. **PHYSICS** The potential energy in joules of a spring that has been stretched or compressed is given by $p(x) = \frac{cx^2}{2}$, where c is the spring constant and x is the distance from equilibrium. When x is negative, the spring is compressed, and when x is positive, the spring is stretched.



a. Describe the transformation(s) of the parent function $f(x) = x^2$ used to graph $p(x)$.

b. The graph of the potential energy for a second spring passes through the point (3, 315). Find the spring constant for the spring and write the function for the potential energy.

Write and graph the function with the given parent function and characteristics.

49. $f(x) = \frac{1}{x}$; expanded vertically by a factor of 2; translated 7 units to the left and 5 units up

50. $f(x) = [\![x]\!]$; expanded vertically by a factor of 3; reflected in the x-axis; translated 4 units down

PHYSICS **The distance an object travels as a function of time is given by $f(t) = \frac{1}{2}at^2 + v_0t + x_0$, where a is the acceleration, v_0 is the initial velocity, and x_0 is the initial position of the object. Describe the transformations of the parent function $f(t) = t^2$ used to graph $f(t)$ for each of the following.**

51. $a = 2, v_0 = 2, x_0 = 0$

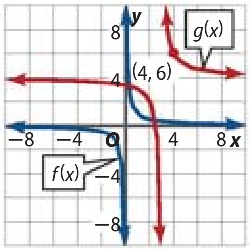
52. $a = 2, v_0 = 0, x_0 = 10$

53. $a = 4, v_0 = 8, x_0 = 1$

54. $a = 3, v_0 = 5, x_0 = 3$

Write an equation for each $g(x)$.

55.



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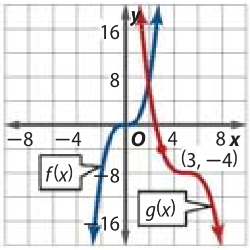

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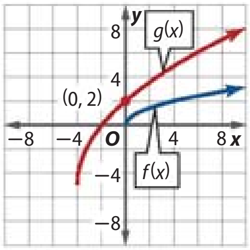

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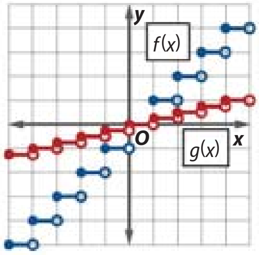


59. **SHOPPING** The management of a new shopping mall originally predicted that attendance in thousands would follow $f(x) = \sqrt{7x}$ for the first 60 days of operation, where x is the number of days after opening and $x = 1$ corresponds with opening day. Write $g(x)$ in terms of $f(x)$ for each situation below.

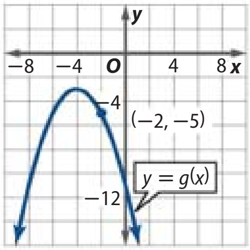
a. Attendance was consistently 12% higher than expected.

b. The opening was delayed 30 days due to construction.

c. Attendance was consistently 450 less than expected.

Identify the parent function $f(x)$ of $g(x)$, and describe the transformation of $f(x)$ used to graph $g(x)$.

60.



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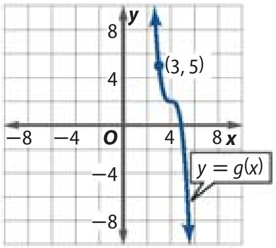

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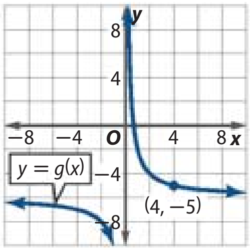

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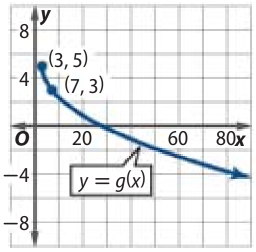
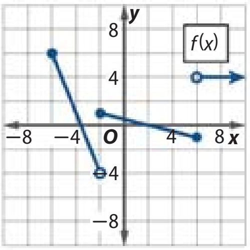


Use $f(x)$ to graph $g(x)$.



64. $g(x) = 0.25f(x) + 4$

65. $g(x) = 3f(x) - 6$

66. $g(x) = f(x - 5) + 3$

67. $g(x) = -2f(x) + 1$

Use $f(x) = \dfrac{8}{\sqrt{x + 6}} - 4$ to graph each function.

68. $g(x) = 2f(x) + 5$

69. $g(x) = -3f(x) + 6$

70. $g(x) = f(4x) - 5$

71. $g(x) = f(2x + 1) + 8$

72. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate operations with functions. Consider

- $f(x) = x^2 + 2x + 7$,
- $g(x) = 4x + 3$, and
- $h(x) = x^2 + 6x + 10$.

a. **TABULAR** Copy and complete the table below for three values of a.

a	$f(a)$	$g(a)$	$f(a) + g(a)$	$h(a)$

b. **VERBAL** How are $f(x)$, $g(x)$, and $h(x)$ related?

c. **ALGEBRAIC** Prove the relationship from part **b** algebraically.

H.O.T. Problems Use Higher-Order Thinking Skills

73. **ERROR ANALYSIS** Danielle and Miranda are describing the transformation $g(x) = [\![x + 4]\!]$. Danielle says that the graph is shifted 4 units to the left, while Miranda says that the graph is shifted 4 units up. Is either of them correct? Explain.

74. **REASONING** Let $f(x)$ be an odd function. If $g(x)$ is a reflection of $f(x)$ in the x-axis and $h(x)$ is a reflection of $g(x)$ in the y-axis, what is the relationship between $f(x)$ and $h(x)$? Explain.

75. **WRITING IN MATH** Explain why order is important when transforming a function with reflections and translations.

REASONING **Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain your reasoning.**

76. If $f(x)$ is an even function, then $f(x) = |f(x)|$.

77. If $f(x)$ is an odd function, then $f(-x) = -|f(x)|$.

78. If $f(x)$ is an even function, then $f(-x) = -|f(x)|$.

79. **CHALLENGE** Describe the transformation of $f(x) = \sqrt{x}$ if $(-2, -6)$ lies on the curve.

80. **REASONING** Suppose (a, b) is a point on the graph of $f(x)$. Describe the difference between the transformations of (a, b) when the graph of $f(x)$ is expanded vertically by a factor of 4 and when the graph of $f(x)$ is compressed horizontally by a factor of 4.

81. **WRITING IN MATH** Use words, graphs, tables, and equations to relate parent functions and transformations. Show this relationship through a specific example.

Spiral Review

Find the average rate of change of each function on the given interval. (Lesson 1-4)

82. $g(x) = -2x^2 + x - 3$; $[-1, 3]$

83. $g(x) = x^2 - 6x + 1$; $[4, 8]$

84. $f(x) = -2x^3 - x^2 + x - 4$; $[-2, 3]$

Use the graph of each function to describe its end behavior. Support the conjecture numerically. (Lesson 1-3)

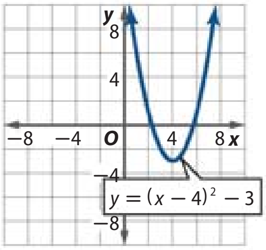
85. $q(x) = -\dfrac{12}{x}$

86. $f(x) = \dfrac{0.5}{x^2}$

87. $p(x) = \dfrac{x+2}{x-3}$

Use the graph of each function to estimate its y-intercept and zero(s). Then find these values algebraically. (Lesson 1-2)

88.



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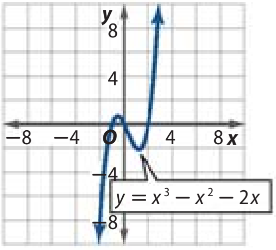

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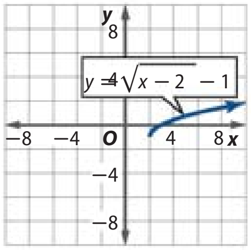


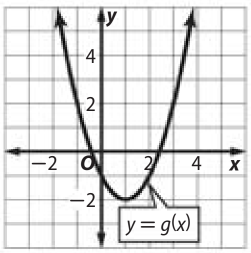
91. GOVERNMENT The number of times each of the first 42 presidents vetoed bills are listed below. What is the standard deviation of the data? (Lesson 0-8)

2, 0, 0, 7, 1, 0, 12, 1, 0, 10, 3, 0, 0, 9,
7, 6, 29, 93, 13, 0, 12, 414, 44, 170, 42, 82, 39, 44,
6, 50, 37, 635, 250, 181, 21, 30, 43, 66, 31, 78, 44, 25

92. LOTTERIES In a multi-state lottery, the player must guess which five of the white balls numbered from 1 to 49 will be drawn. The order in which the balls are drawn does not matter. The player must also guess which one of the red balls numbered from 1 to 42 will be drawn. How many ways can the player complete a lottery ticket? (Lesson 0-7)

Skills Review for Standardized Tests

93. SAT/ACT The figure shows the graph of $y = g(x)$, which has a minimum located at $(1, -2)$. What is the maximum value of $h(x) = -3g(x) - 1$?



A 6

B 5

C 3

D 2

E It cannot be determined from the information given.

94. REVIEW What is the simplified form of $\dfrac{4x^3y^2z^{-1}}{(x^{-2}y^3z^2)^2}$?

95. What is the range of $y = \dfrac{x^2+8}{2}$?

F $\{y \mid y \neq \pm 2\sqrt{2}\}$

G $\{y \mid y \geq 4\}$

H $\{y \mid y \geq 0\}$

J $\{y \mid y \leq 0\}$

96. REVIEW What is the effect on the graph of $y = kx^2$ as k decreases from 3 to 2?

A The graph of $y = 2x^2$ is a reflection of the graph of $y = 3x^2$ across the y-axis.

B The graph is rotated 90° about the origin.

C The graph becomes narrower.

D The graph becomes wider.

EXTEND 1-5

Graphing Technology Lab
Nonlinear Inequalities

Objective

- Use a graphing calculator to solve nonlinear inequalities.

A nonlinear inequality in one variable can be solved graphically by converting it into two inequalities in two variables and finding the intersection. You can use a graphing calculator to find this intersection.

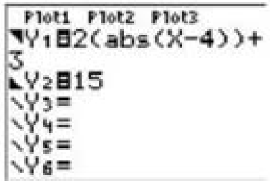
Activity Solve an Inequality by Graphing

Solve $2|x - 4| + 3 < 15$.

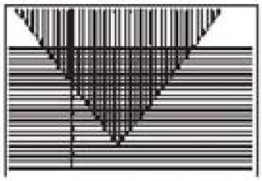
Step 1 Separate this inequality into two inequalities, one for each side of the inequality symbol. Replace each side with y to form the new inequalities.

$2|x - 4| + 3 <$ Y1; Y2 < 15

Step 2 Graph each inequality. Go to the left of the equals sign and select ENTER until the shaded triangles flash to make each inequality sign. The triangle above represents *greater than* and the triangle below represents *less than*. For abs(, press MATH ▶ 1.



Step 3 Graph the inequalities in the appropriate window. Either use the ZOOM feature or adjust the window manually to display both graphs. Any window that shows the two intersection points will work.



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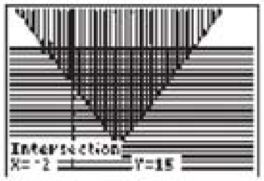
Step 4 The darkly shaded area indicates the intersection of the graphs and the solution of the system of inequalities. Use the intersection feature to find that the two graphs intersect at (−2, 15) and (10, 15).



[−5, 15] scl: 1 by [0, 20] scl: 1

Step 5 The solution occurs in the region of the graph where $-2 < x < 10$. Thus, the solution to $2|x - 4| + 3 < 15$ is the set of x-values such that $-2 < x < 10$. Check your solution algebraically by confirming that an x-value in this interval is a solution of the inequality.

StudyTip

Adjusting the Window You can use the ZoomFit or ZoomOut options or manually adjust the window to include both graphs.

Exercises

Solve each inequality by graphing.

1. $3|x + 2| - 4 > 8$

2. $-2|x + 4| + 6 \leq 2$

3. $5|2x + 1| > 15$

4. $-3|2x - 3| + 1 \leq 10$

5. $|x - 6| > x + 2$

6. $|2x + 1| \geq 4x - 3$

Extension

7. REASONING Describe the appearance of the graph for an inequality with no solution.

8. CHALLENGE Solve $-10x - 32 < |x + 3| - 2 < -|x + 4| + 8$ by graphing.

LESSON

1-6 Function Operations and Composition of Functions

Then

- You evaluated functions. (Lesson 1-1)

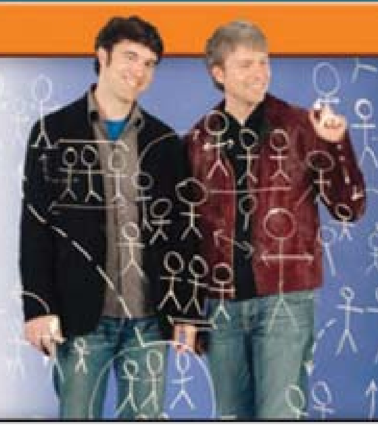
Now

1. Perform operations with functions.
2. Find compositions of functions.

Why?

In April 2008, the top social networking site, founded by Chris DeWolfe and Tom Anderson, had over 60.4 million unique visitors. The number two site at that time had 24.9 million unique visitors, or 35.5 million fewer visitors.

Suppose $A(t)$ and $B(t)$ model the number of unique visitors to the number one and two social networking sites, respectively, t years since 2000. $A(t) - B(t)$ represents the difference in the number of unique visitors between the two sites for t years after 2000.



NewVocabulary
composition

1 Operations with Functions

Just as you can combine two real numbers using addition, subtraction, multiplication, and division, you can combine two functions.

KeyConcept Operations with Functions

Let f and g be two functions with intersecting domains. Then for all x-values in the intersection, the sum, product, difference, and quotient of f and g are new functions defined as follows.

Sum $(f + g)(x) = f(x) + g(x)$

Product $(f \cdot g)(x) = f(x) \cdot g(x)$

Difference $(f - g)(x) = f(x) - g(x)$

Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

For each new function, the domain consists of those values of x common to the domains of f and g. The domain of the quotient function is further restricted by excluding any values that make the denominator, $g(x)$, zero.

Example 1 Operations with Functions

Given $f(x) = x^2 + 4x$, $g(x) = \sqrt{x+2}$, and $h(x) = 3x - 5$, find each function and its domain.

a. $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$
$$= (x^2 + 4x) + \left(\sqrt{x+2}\right)$$
$$= x^2 + 4x + \sqrt{x+2}$$

The domain of f is $(-\infty, \infty)$, and the domain of g is $[-2, \infty)$. So, the domain of $(f + g)$ is the intersection of these domains or $[-2, \infty)$.

b. $(f - h)(x)$

$$(f - h)(x) = f(x) - h(x)$$
$$= (x^2 + 4x) - (3x - 5)$$
$$= x^2 + 4x - 3x + 5$$
$$= x^2 + x + 5$$

The domains of f and h are both $(-\infty, \infty)$, so the domain of $(f - h)$ is $(-\infty, \infty)$.

c. $(f \cdot h)(x)$

$$(f \cdot h)(x) = f(x) \cdot h(x)$$
$$= (x^2 + 4x)(3x - 5)$$
$$= 3x^3 - 5x^2 + 12x^2 - 20x$$
$$= 3x^3 + 7x^2 - 20x$$

The domains of f and h are both $(-\infty, \infty)$, so the domain of $(f \cdot h)$ is $(-\infty, \infty)$.

d. $\left(\frac{h}{f}\right)(x)$

$$\left(\frac{h}{f}\right)(x) = \frac{h(x)}{f(x)} \text{ or } \frac{3x-5}{x^2+4x}$$

The domain of h and f are both $(-\infty, \infty)$, but $x = 0$ or $x = -4$ yields a zero in the denominator of $\left(\frac{h}{f}\right)$. So, the domain of $\left(\frac{h}{f}\right)$ is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

GuidedPractice

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

1A. $f(x) = x - 4$, $g(x) = \sqrt{9 - x^2}$

1B. $f(x) = x^2 - 6x - 8$, $g(x) = \sqrt{x}$

2 Composition of Functions

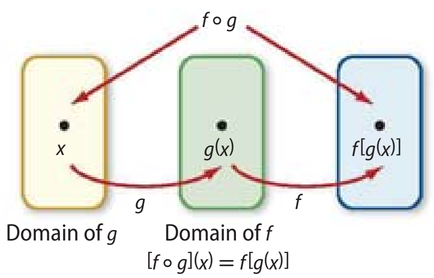
The function $y = (x - 3)^2$ combines the linear function $y = x - 3$ with the squaring function $y = x^2$, but the combination does not involve addition, subtraction, multiplication, or division. This combining of functions, called *composition*, is the result of one function being used to evaluate a second function.

KeyConcept Composition of Functions

The **composition** of function f with function g is defined by

$$[f \circ g](x) = f[g(x)].$$

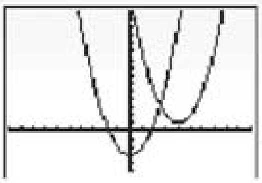
The domain of $f \circ g$ includes all x-values in the domain of g that map to $g(x)$-values in the domain of g as shown.



In the composition $f \circ g$, which is read as *f composition g* or *f of g*, the function g is applied first and then f.

WatchOut!

Order of Composition In most cases, $g \circ f$ and $f \circ g$ are different functions. That is, composition of functions is not commutative. Notice that the graphs of $[f \circ g](x) = x^2 - 8x + 17$ and $[g \circ f](x) = x^2 - 3$ from Example 2 are different.



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Example 2 Compose Two Functions

Given $f(x) = x^2 + 1$ and $g(x) = x - 4$, find each of the following.

a. $[f \circ g](x)$

$[f \circ g](x) = f[g(x)]$ — Definition of $f \circ g$

$= f(x - 4)$ — Replace $g(x)$ with $x - 4$.

$= (x - 4)^2 + 1$ — Substitute $x - 4$ for x in $f(x)$.

$= x^2 - 8x + 16 + 1$ or $x^2 - 8x + 17$ — Simplify.

b. $[g \circ f](x)$

$[g \circ f](x) = g[f(x)]$ — Definition of $g \circ f$

$= g(x^2 + 1)$ — Replace $f(x)$ with $x^2 + 1$.

$= (x^2 + 1) - 4$ or $x^2 - 3$ — Substitute $x^2 + 1$ for x in $g(x)$.

c. $[f \circ g](2)$

Evaluate the expression $[f \circ g](x)$ you wrote in part **a** for $x = 2$.

$[f \circ g](2) = (2)^2 - 8(2) + 17$ or 5 — Substitute 2 for x in $x^2 - 8x + 17$.

GuidedPractice

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](3)$.

2A. $f(x) = 3x + 1$, $g(x) = 5 - x^2$

2B. $f(x) = 6x^2 - 4$, $g(x) = x + 2$

Because the domains of f and g in Example 2 include all real numbers, the domain of $f \circ g$ is all real numbers, $\mathbb{R}$.

When the domains of f or g are restricted, the domain of $f \circ g$ is restricted to all x-values in the domain of g whose range values, $g(x)$, are in the domain of f.

Example 3 Find a Composite Function with a Restricted Domain

Find $f \circ g$.

a. $f(x) = \frac{1}{x+1}$, $g(x) = x^2 - 9$

StudyTip

Domains of Composite Functions It is very important to complete the domain analysis before performing the composition. Domain restrictions may not be evident after the composition is simplified.

To find $f \circ g$, you must first be able to find $g(x) = x^2 - 9$, which can be done for all real numbers. Then you must be able to evaluate $f(x) = \frac{1}{x+1}$ for each of these $g(x)$-values, which can only be done when $g(x) \neq -1$. Excluding from the domain those values for which $x^2 - 9 = -1$, namely when $x = \pm\sqrt{8}$ or $\pm 2\sqrt{2}$, the domain of $f \circ g$ is $\{x \mid x \neq \pm 2\sqrt{2}, x \in \mathbb{R}\}$.

Now find $[f \circ g](x)$.

$[f \circ g](x) = f[g(x)]$ — Definition of $f \circ g$

$= f(x^2 - 9)$ — Replace $g(x)$ with $x^2 - 9$.

$= \frac{1}{x^2 - 9 + 1}$ or $\frac{1}{x^2 - 8}$ — Substitute $x^2 - 9$ for x in $f(x)$.

Notice that $\frac{1}{x^2 - 8}$ is undefined when $x^2 - 8 = 0$, which is when $x = \pm 2\sqrt{2}$. Because the implied domain is the same as the domain determined by considering the domains of f and g, the composition can be written as $[f \circ g](x) = \frac{1}{x^2 - 8}$ for $x \neq \pm 2\sqrt{2}$.

b. $f(x) = x^2 - 2$, $g(x) = \sqrt{x - 3}$

To find $f \circ g$, you must first be able to find $g(x)$, which can only be done for $x \geq 3$. Then you must be able to square each of these $g(x)$-values and subtract 2, which can be done for all real numbers. Therefore, the domain of $f \circ g$ is $\{x \mid x \geq 3, x \in \mathbb{R}\}$. Now find $[f \circ g](x)$.

$[f \circ g](x) = f[g(x)]$ — Definition of $f \circ g$

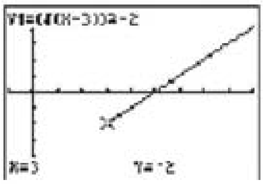
$= f\left(\sqrt{x - 3}\right)$ — Replace $g(x)$ with $\sqrt{x - 3}$.

$= \left(\sqrt{x - 3}\right)^2 - 2$ — Substitute $\sqrt{x - 3}$ for x in $f(x)$.

$= x - 3 - 2$ or $x - 5$ — Simplify.

Once the composition is simplified, it appears that the function is defined for all reals, which is known to be untrue. Therefore, write the composition as $[f \circ g](x) = x - 5$ for $x \geq 3$.

CHECK Use a graphing calculator to check this result. Enter the function as $y = (\sqrt{x - 3})^2 - 2$. The graph appears to be part of the line $y = x - 5$. Then use the TRACE feature to help determine that the domain of the composite function begins at $x = 3$ and extends to ∞.



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StudyTip

Using Absolute Value Recall from Lesson 0-4 that when you find an even root of an even power and the result is an odd power, you must use the absolute value of the result to ensure that the answer is nonnegative. For example, $\sqrt{x^2} = |x|$.

GuidedPractice

3A. $f(x) = \sqrt{x + 1}$, $g(x) = x^2 - 1$

3B. $f(x) = \frac{5}{x}$, $g(x) = x^2 + x$

An important skill in calculus is to be able to *decompose* a function into two simpler functions. To decompose a function h, find two functions with a composition of h.

Example 4 Decompose a Composite Function

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$.

a. $h(x) = \sqrt{x^3 - 4}$

Observe that h is defined using the square root of $x^3 - 4$. So one way to write h as a composition of two functions is to let $g(x) = x^3 - 4$ and $f(x) = \sqrt{x}$. Then

$$h(x) = \sqrt{x^3 - 4} = \sqrt{g(x)} = f[g(x)] \text{ or } [f \circ g](x).$$

b. $h(x) = 2x^2 + 20x + 50$

$h(x) = 2x^2 + 20x + 50$ — Notice that $h(x)$ is factorable.

$= 2(x^2 + 10x + 25)$ or $2(x + 5)^2$ — Factor.

One way to write $h(x)$ as a composition is to let $f(x) = 2x^2$ and $g(x) = x + 5$.

$$h(x) = 2(x + 5)^2 = 2[g(x)]^2 = f[g(x)] \text{ or } [f \circ g](x).$$

GuidedPractice

4A. $h(x) = x^2 - 2x + 1$

4B. $h(x) = \frac{1}{x + 7}$

Real-WorldCareer

Computer Animator
Animators work in many industries to create the animated images used in movies, television, and video games. Computer animators must be artistic, and most have received post-secondary training at specialized schools.

You can use the composition of functions to solve real-world problems.

Real-World Example 5 Compose Real-World Functions

COMPUTER ANIMATION **To animate the approach of an opponent directly in front of a player, a computer game animator starts with an image of a 20-pixel by 60-pixel rectangle. The animator then increases each dimension of the rectangle by 15 pixels per second.**

a. Find functions to model the data.

The length L of the rectangle increases at a rate of 15 pixels per second, so $L(t) = 20 + 15t$, where t is the time in seconds and $t \geq 0$. The area of the rectangle is its length L times its width. The width is 40 pixels more than its length or $L + 40$. So, the area of the rectangle is $A(L) = L(L + 40)$ or $L^2 + 40L$ and $L \geq 20$.

b. Find $A \circ L$. What does this function represent?

$A \circ L = A[\mathbf{L(t)}]$ — Definition of $A \circ L$

$= A(\mathbf{20 + 15t})$ — Replace $L(t)$ with $20 + 15t$.

$= (\mathbf{20 + 15t})^2 + 40(\mathbf{20 + 15t})$ — Substitute $20 + 15t$ for L in $A(L)$.

$= 225t^2 + 1200t + 1200$ — Simplify.

This composite function models the area of the rectangle as a function of time.

c. How long does it take for the rectangle to triple its original size?

The initial area of the rectangle is $20 \cdot 60$ or 1200 pixels. The rectangle will be three times its original size when $[A \circ L](t) = 225t^2 + 1200t + 1200 = 3600$. Solve for t to find that $t \approx 1.55$ or -6.88. Because a negative t-value is not part of the domain of $L(t)$, it is also not part of the domain of the composite function. The area will triple after about 1.55 seconds.

GuidedPractice

5. **BUSINESS** A computer store offers a 15% discount to college students on the purchase of any notebook computer. The store also advertises $100 coupons.

A. Find functions to model the data.

B. Find $[c \circ d](x)$ and $[d \circ c](x)$. What does each composite function represent?

C. Which composition of the coupon and discount results in the lower price? Explain.

Exercises

= Step-by-Step Solutions begin on page R29.

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function. (Example 1)

1. $f(x) = x^2 + 4$, $g(x) = \sqrt{x}$

2. $f(x) = 8 - x^3$, $g(x) = x - 3$

3. $f(x) = x^2 + 5x + 6$, $g(x) = x + 2$

4. $f(x) = x - 9$, $g(x) = x + 5$

5. $f(x) = x^2 + x$, $g(x) = 9x$

6. $f(x) = x - 7$, $g(x) = x + 7$

7. $f(x) = \frac{6}{x}$, $g(x) = x^3 + x$

8. $f(x) = \frac{x}{4}$, $g(x) = \frac{3}{x}$

9. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = 4\sqrt{x}$

10. $f(x) = \frac{3}{x}$, $g(x) = x^4$

11. $f(x) = \sqrt{x+8}$, $g(x) = \sqrt{x+5} - 3$

12. $f(x) = \sqrt{x+6}$, $g(x) = \sqrt{x-4}$

13. BUDGETING Suppose a budget in dollars for one person for one month is approximated by $f(x) = 25x + 350$ and $g(x) = 15x + 200$, where f is the cost of rent and groceries, g is the cost of gas and all other expenses, and $x = 1$ represents the end of the first week. (Example 1)

a. Find $(f+g)(x)$ and the relevant domain.

b. What does $(f+g)(x)$ represent?

c. Find $(f+g)(4)$. What does this value represent?

14. PHYSICS Two different forces act on an object being pushed across a floor: the force of the person pushing the object and the force of friction. If W is work in joules, F is force in newtons, and d is displacement of the object in meters, $W_p(d) = F_p d$ describes the work of the person and $W_f(d) = F_f d$ describes the work done by friction. The increase in kinetic energy of the object is the difference between the work done by the person W_p and the work done by friction W_f. (Example 1)

a. Find $(W_p - W_f)(d)$.

b. Determine the net work expended when a person pushes a box 50 meters with a force of 95 newtons and friction exerts a force of 55 newtons.

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](6)$. (Example 2)

15. $f(x) = 2x - 3$, $g(x) = 4x - 8$

16. $f(x) = -2x^2 - 5x + 1$, $g(x) = -5x + 6$

17. $f(x) = 8 - x^2$, $g(x) = x^2 + x + 1$

18. $f(x) = x^2 - 16$, $g(x) = x^2 + 7x + 11$

19. $f(x) = 3 - x^2$, $g(x) = x^3 + 1$

20. $f(x) = 2 + x^4$, $g(x) = -x^2$

Find $f \circ g$. (Example 3)

21. $f(x) = \frac{1}{x+1}$, $g(x) = x^2 - 4$

22. $f(x) = \frac{2}{x-3}$, $g(x) = x^2 + 6$

23. $f(x) = \sqrt{x+4}$, $g(x) = x^2 - 4$

24. $f(x) = x^2 - 9$, $g(x) = \sqrt{x+3}$

25. $f(x) = \frac{5}{x}$, $g(x) = \sqrt{6-x}$

26. $f(x) = -\frac{4}{x}$, $g(x) = \sqrt{x+8}$

27. $f(x) = \sqrt{x+5}$, $g(x) = x^2 + 4x - 1$

28. $f(x) = \sqrt{x-2}$, $g(x) = x^2 + 8$

29. RELATIVITY In the theory of relativity, $m(v) = \frac{100}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is the speed of light, 300 million meters per second, and m is the mass of a 100-kilogram object at speed v in meters per second. (Example 4)

a. Are there any restrictions on the domain of the function? Explain their meaning.

b. Find $m(10)$, $m(10{,}000)$, and $m(1{,}000{,}000)$.

c. Describe the behavior of $m(v)$ as v approaches c.

d. Decompose the function into two separate functions.

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$. (Example 4)

30. $h(x) = \sqrt{4x+2} + 7$

31. $h(x) = \frac{6}{x+5} - 8$

32. $h(x) = |4x + 8| - 9$

33. $h(x) = [\![-3(x-9)]\!]$

34. $h(x) = \sqrt{\frac{5-x}{x+2}}$

35. $h(x) = (\sqrt{x} + 4)^3$

36. $h(x) = \frac{6}{(x+2)^2}$

37. $h(x) = \frac{8}{(x-5)^2}$

38. $h(x) = \frac{\sqrt{4+x}}{x-2}$

39. $h(x) = \frac{x+5}{\sqrt{x} - 1}$

40. QUANTUM MECHANICS The wavelength λ of a particle with mass m kilograms moving at v meters per second is represented by $\lambda = \frac{h}{mv}$, where h is a constant equal to $6.626 \cdot 10^{-34}$.

a. Find a function to represent the wavelength of a 25-kilogram object as a function of its speed.

b. Are there any restrictions on the domain of the function? Explain their meaning.

c. If the object is traveling 8 meters per second, find the wavelength in terms of h.

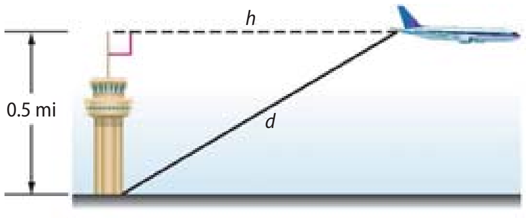
d. Decompose the function into two separate functions.

41. **JOBS** A salesperson for an insurance agency is paid an annual salary plus a bonus of 4% of sales made over \$300,000. Let $f(x) = x - \$300{,}000$ and $h(x) = 0.04x$, where x is total sales. (Example 5)

a. If x is greater than \$300,000, is the bonus represented by $f[h(x)]$ or by $h[f(x)]$? Explain your reasoning.

b. Determine the amount of bonus for one year with sales of \$450,000.

42. **TRAVEL** An airplane flying above a landing strip at 275 miles per hour passes a control tower 0.5 mile below at time $t = 0$ hours. (Example 5)



a. Find the distance d between the airplane and the control tower as a function of the horizontal distance h from the control tower to the plane.

b. Find h as a function of time t.

c. Find $d \circ h$. What does this function represent?

d. If the plane continued to fly the same distance from the ground, how far would the plane be from the control tower after 10 minutes?

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$.

43. $h(x) = \sqrt{x - 1} - \frac{4}{x}$

44. $h(x) = \sqrt{2x + 6} + \frac{6}{x}$

45. $h(x) = \frac{8}{x^2 + 2} + 5|x|$

46. $h(x) = \sqrt{-7x} + 9x$

47. $h(x) = \frac{x}{2x - 1} + \sqrt{\frac{4}{x}}$

48. $h(x) = \frac{x^2 - 4}{x} + \frac{3x - 5}{5x}$

Use the given information to find $f(0.5)$, $f(-6)$, and $f(x + 1)$. Round to the nearest tenth if necessary.

49. $f(x) - g(x) = x^2 + x - 6$, $g(x) = x + 4$

50. $f(x) + g(x) = \frac{2}{x^2} + \frac{1}{x} - \frac{1}{3}$, $g(x) = 2x$

51. $g(x) - f(x) + \frac{3}{5} = 9x^2 + 4x$, $g(x) = \frac{x}{10}$

52. $g(x) = f(x) - 18x^2 + \frac{\sqrt{2}}{x}$, $g(x) = \sqrt{1 - x}$

Find $[f \circ g \circ h](x)$.

53. $f(x) = x + 8$
$g(x) = x^2 - 6$
$h(x) = \sqrt{x} + 3$

54. $f(x) = x^2 - 2$
$g(x) = 5x + 12$
$h(x) = \sqrt{x} - 4$

55. $f(x) = \sqrt{x + 5}$
$g(x) = x^2 - 3$
$h(x) = \frac{1}{x}$

56. $f(x) = \frac{3}{x}$
$g(x) = x^2 - 4x + 1$
$h(x) = x + 2$

57. If $f(x) = x + 2$, find $g(x)$ such that:

a. $(f + g)(x) = x^2 + x + 6$

b. $\left(\frac{f}{g}\right)(x) = \frac{1}{4}$

58. If $f(x) = \sqrt{4x}$, find $g(x)$ such that:

a. $[f \circ g](x) = |6x|$

b. $[g \circ f](x) = 200x + 25$

59. If $f(x) = 4x^2$, find $g(x)$ such that:

a. $(f \cdot g)(x) = x$

b. $(f \cdot g)(x) = \frac{1}{8}x^{\frac{7}{3}}$

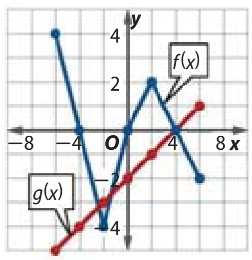
60. **INTEREST** An investment account earns interest compounded quarterly. If x dollars are invested in an account, the investment $I(x)$ after one quarter is the initial investment plus accrued interest or $I(x) = 1.016x$.

a. Find $[I \circ I](x)$, $[I \circ I \circ I](x)$, and $[I \circ I \circ I \circ I](x)$.

b. What do the compositions represent?

c. What is the account's annual percentage yield?

Use the graphs of $f(x)$ and $g(x)$ to find each function value.



61. $(f + g)(2)$

62. $(f - g)(-6)$

63. $(f \cdot g)(4)$

64. $\left(\frac{f}{g}\right)(-2)$

65. $[f \circ g](-4)$

66. $[g \circ f](6)$

67. **CHEMISTRY** The average speed $v(m)$ of gas molecules at 30°C in meters per second can be represented by $v(m) = \sqrt{\frac{(24.9435)(303)}{m}}$, where m is the molar mass of the gas in kilograms per mole.

a. Are there any restrictions on the domain of the function? Explain their meaning.

b. Find the average speed of 145 kilograms per mole gas molecules at 30°C.

c. How will the average speed change as the molar mass of gas increases?

d. Decompose the function into two separate functions.

Find functions f, g, and h such that $a(x) = [f \circ g \circ h](x)$.

68. $a(x) = (\sqrt{x - 7} + 4)^2$

69. $a(x) = \sqrt{(x - 5)^2 + 8}$

70. $a(x) = \frac{3}{(x - 3)^2 + 4}$

71. $a(x) = \frac{4}{(\sqrt{x} + 3)^2 + 1}$

For each pair of functions, find $f \circ g$ and $g \circ f$.

72. $f(x) = x^2 - 6x + 5$
$g(x) = \sqrt{x + 4} + 3$

73. $f(x) = x^2 + 8x - 3$
$g(x) = \sqrt{x + 19} - 4$

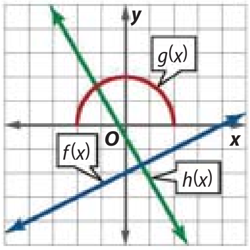
74. $f(x) = \sqrt{x + 6}$
$g(x) = \sqrt{16 + x^2}$

75. $f(x) = \sqrt{x}$
$g(x) = \sqrt{9 - x^2}$

76. $f(x) = -\frac{8}{5 - 4x}$
$g(x) = \frac{2}{3 + x}$

77. $f(x) = \frac{6}{2x + 1}$
$g(x) = \frac{4}{4 - x}$

Graph each of the following.



78. $(f + h)(x)$

79. $(h - f)(x)$

80. $(f + g)(x)$

81. $(h + g)(x)$

82. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate inverses of functions.

a. ALGEBRAIC Find the composition of f with g and of g with f for each pair of functions.

$f(x)$	$g(x)$
$x + 3$	$x - 3$
$4x$	$\frac{x}{4}$
x^3	$\sqrt[3]{x}$

b. VERBAL Describe the relationship between the composition of each pair of functions.

c. GRAPHICAL Graph each pair of functions on the coordinate plane. Graph the line of reflection by finding the midpoint of the segment between corresponding points.

d. VERBAL Make a conjecture about the line of reflection between the functions.

e. ANALYTICAL The compositions $[f \circ g](x)$ and $[g \circ f](x)$ are equivalent to which parent function?

f. ANALYTICAL Find $g(x)$ for each $f(x)$ such that $[f \circ g](x) = [g \circ f](x) = x$.

i. $f(x) = x - 6$

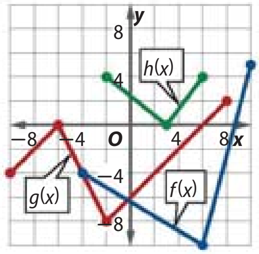
ii. $f(x) = \frac{x}{3}$

iii. $f(x) = x^5$

iv. $f(x) = 2x - 3$

v. $f(x) = x^3 + 1$

State the domain of each composite function.



83. $[f \circ g](x)$

84. $[g \circ f](x)$

85. $[h \circ f](x)$

86. $[h \circ g](x)$

H.O.T. Problems Use Higher-Order Thinking Skills

REASONING **Determine whether $[f \circ g](x)$ is *even, odd, neither,* or *not enough information* for each of the following.**

87. f and g are odd.

88. f and g are even.

89. f is even and g is odd.

90. f is odd and g is even.

CHALLENGE **Find a function f other than $f(x) = x$ such that the following are true.**

91. $[f \circ f](x) = x$

92. $[f \circ f \circ f](x) = f(x)$

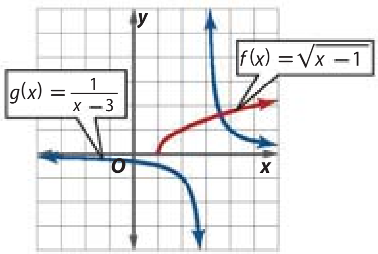
93. **WRITING IN MATH** Explain how $f(x)$ might have a domain restriction while $[f \circ g](x)$ might not. Provide an example to justify your reasoning.

94. **REASONING** Determine whether the following statement is *true* or *false*. Explain your reasoning.

If f is a square root function and g is a quadratic function, then $f \circ g$ is always a linear function.

95 **CHALLENGE** State the domain of $[f \circ g \circ h](x)$ for $f(x) = \frac{1}{x - 2}$, $g(x) = \sqrt{x + 1}$, and $h(x) = \frac{4}{x}$.

96. **WRITING IN MATH** Describe how you would find the domain of $[f \circ g](x)$.



97. **WRITING IN MATH** Explain why order is important when finding the composition of two functions.

Spiral Review

98. FINANCIAL LITERACY The cost of labor for servicing cars at B & B Automotive is displayed in the advertisement. (Lesson 1-5)

a. Graph the function that describes the cost for x hours of labor.

b. Graph the function that would show a $25 additional charge if you decide to also get the oil changed and fluids checked.

c. What would be the cost of servicing a car that required 3.45 hours of labor if the owner requested to have the oil changed and the fluids checked?

Approximate to the nearest hundredth the relative or absolute extrema of each function. State the x-values where they occur. (Lesson 1-4)

99. $f(x) = 2x^3 - 3x^2 + 4$

100. $g(x) = -x^3 + 5x - 3$

101. $f(x) = x^4 + x^3 - 2$

Approximate the real zeros of each function for the given interval. (Lesson 1-3)

102. $g(x) = 2x^5 - 2x^4 - 4x^2 - 1$; $[-1, 3]$

103. $f(x) = \frac{x^2 - 3}{x - 4}$; $[-3, 3]$

104. $g(x) = \frac{x^2 - 2x - 1}{x^2 + 3x}$; $[1, 5]$

105. SPORTS The table shows the leading home run and runs batted in totals in the American League for 2004–2008. (Lesson 1-1)

Year	2004	2005	2006	2007	2008
HR	43	48	54	54	48
RBI	150	148	137	156	146

Source: *World Almanac*

a. Make a graph of the data with home runs on the horizontal axis and runs batted in on the vertical axis.

b. Identify the domain and range.

c. Does the graph represent a function? Explain your reasoning.

Skills Review for Standardized Tests

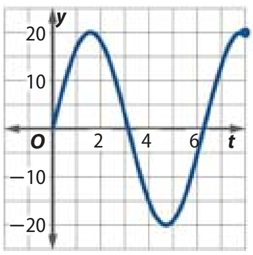
106. SAT/ACT A jar contains only red, green, and blue marbles. It is three times as likely that you randomly pick a red marble as a green marble, and five times as likely that you pick a green one as a blue one. Which could be the number of marbles in the jar?

A 39
B 40
C 41
D 42
E 63

107. If $g(x) = x^2 + 9x + 21$ and $h(x) = 2(x - 5)^2$, then $h[g(x)] =$

F $x^4 + 18x^3 + 113x^2 + 288x + 256$
G $2x^4 + 36x^3 + 226x^2 + 576x + 512$
H $3x^4 + 54x^3 + 339x^2 + 864x + 768$
J $4x^4 + 72x^3 + 452x^2 + 1152x + 1024$

108. FREE RESPONSE The change in temperature of a substance in degrees Celsius as a function of time for $0 \leq t \leq 8$ is shown in the graph.



a. This graph represents a function. Explain why.

b. State the domain and range.

c. If the initial temperature is 25°C, what is the approximate temperature of the substance at $t = 7$?

d. Analyze the graph for symmetry and zeros. Determine if the function is *even*, *odd*, or *neither*.

e. Is the function continuous at $t = 2$? Explain.

f. Determine the intervals on which the function is increasing or decreasing.

g. Estimate the average rate of change for [2, 5].

h. What is the significance of your answers to parts **f** and **g** in the context of the situation?

LESSON 1-7 Inverse Relations and Functions

Then

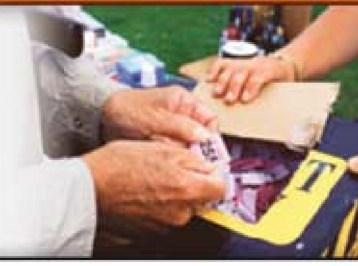
You found the composition of two functions. (Lesson 1-6)

Now

1. Use the horizontal line test to determine inverse functions.
2. Find inverse functions algebraically and graphically.

Why?

The Band Boosters at Julia's high school are selling raffle tickets. Table A relates the cost in dollars to the number of tickets purchased. Table B relates the number of tickets that can be purchased to the number of dollars spent. By interchanging the input and output from Table A, Julia obtains Table B.



NewVocabulary
inverse relation
inverse function
one-to-one

Table A

Tickets	1	2	3	4	6
Cost ($)	2	4	6	8	10

Table B

Money Spent ($)	2	4	6	8	10
Tickets	1	2	3	4	6

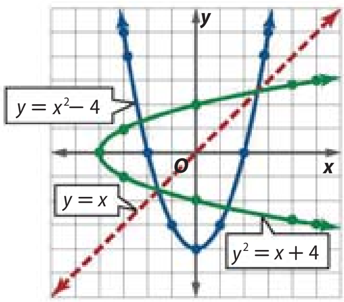
1 Inverse Functions

The relation shown in Table A is the *inverse relation* of the relation shown in Table B. **Inverse relations** exist if and only if one relation contains (b, a) whenever the other relation contains (a, b). When a relation is expressed as an equation, its inverse relation can be found by interchanging the independent and dependent variables. Consider the following.

Relation

$y = x^2 - 4$

x	y
−3	5
−2	0
−1	−3
0	−4
1	−3
2	0
3	5



Inverse Relation

$x = y^2 - 4$ or $y^2 = x + 4$

x	y
5	−3
0	−2
−3	−1
−4	0
−3	1
0	2
5	3

Notice that these inverse relations are reflections of each other in the line $y = x$. This relationship is true for the graphs of all relations and their inverse relations. We are most interested in *functions* with inverse relations that are also *functions*. If the inverse relation of a function f is also a function, then it is called the **inverse function** of f and is denoted f^{-1}, read *f inverse*.

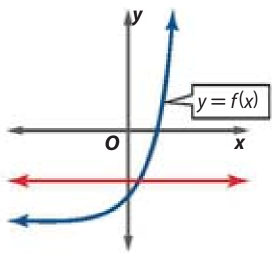
Not all functions have inverse functions. In the graph above, notice that the original relation is a function because it passes the vertical line test. But its inverse relation fails this test, so it is not a function. The reflective relationship between the graph of a function and its inverse relation leads us to the following graphical test for determining whether the inverse function of a function exists.

KeyConcept Horizontal Line Test

Words A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.

Example Since no horizontal line intersects the graph of f more than once, the inverse function f^{-1} exists.

Model



WatchOut!

Horizontal Line Test When using a graphing calculator, closely examine places where it appears that the function may fail the horizontal line test. Use Zoom In and Zoom Out features, or adjust the window to be sure.

Example 1 Apply the Horizontal Line Test

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

a. $f(x) = |x - 1|$

The graph of $f(x)$ in Figure 1.7.1 shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that f^{-1} does not exist.

b. $g(x) = x^3 - 6x^2 + 12x - 8$

The graph of $g(x)$ in Figure 1.7.2 shows that it is not possible to find a horizontal line that intersects the graph of $g(x)$ in more than one point. Therefore, you can conclude that g^{-1} exists.

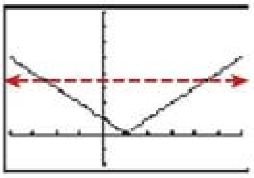


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Figure 1.7.1

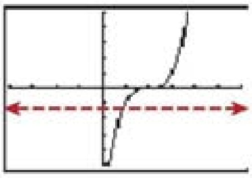

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Figure 1.7.2

GuidedPractice

1A. $h(x) = \frac{4}{x}$

1B. $f(x) = x^2 + 5x - 7$

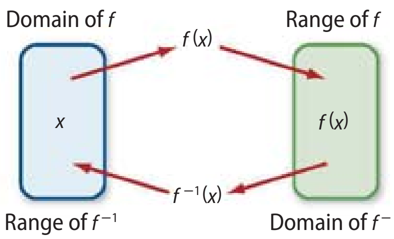
2 Find Inverse Functions

If a function passes the horizontal line test, then it is said to be **one-to-one**, because no x-value is matched with more than one y-value and no y-value is matched with more than one x-value.

ReadingMath

Inverse Function Notation The symbol $f^{-1}(x)$ should not be confused with the reciprocal function $\frac{1}{f(x)}$. If f is a function, the symbol f^{-1} can only be interpreted as f inverse of x.

If a function f is one-to-one, it has an inverse function f^{-1} such that the domain of f is equal to the range of f^{-1}, and the range of f is equal to the domain of f^{-1}.



To find an inverse function algebraically, follow the steps below.

KeyConcept Finding an Inverse Function

Step 1 Determine whether the function has an inverse by checking to see if it is one-to-one using the horizontal line test.

Step 2 In the equation for $f(x)$, replace $f(x)$ with y and then interchange x and y.

Step 3 Solve for y and then replace y with $f^{-1}(x)$ in the new equation.

Step 4 State any restrictions on the domain of f^{-1}. Then show that the domain of f is equal to the range of f^{-1} and the range of f is equal the domain of f^{-1}.

The last step implies that only part of the function you find algebraically may be the inverse function of f. Therefore, be sure to analyze the domain of f when finding f^{-1}.

ReadingMath

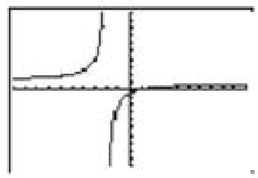
Invertible Functions A function that has an inverse function is said to be *invertible*.

Example 2 Find Inverse Functions Algebraically

Determine whether f has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

a. $f(x) = \dfrac{x-1}{x+2}$

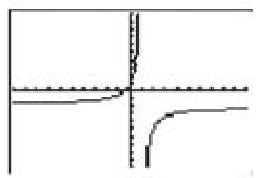
The graph of f shown passes the horizontal line test. Therefore, f is a one-to-one function and has an inverse function. From the graph, you can see that f has domain $(-\infty, -2) \cup (-2, \infty)$ and range $(-\infty, 1) \cup (1, \infty)$. Now find f^{-1}.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

$f(x) = \dfrac{x-1}{x+2}$	Original function
$y = \dfrac{x-1}{x+2}$	Replace $f(x)$ with y.
$x = \dfrac{y-1}{y+2}$	Interchange x and y.
$xy + 2x = y - 1$	Multiply each side by $y + 2$. Then apply the Distributive Property.
$xy - y = -2x - 1$	Isolate the y-terms.
$y(x-1) = -2x - 1$	Distributive Property
$y = \dfrac{-2x-1}{x-1}$	Solve for y.
$f^{-1}(x) = \dfrac{-2x-1}{x-1}$	Replace y with $f^{-1}(x)$. Note that $x \neq 1$.

From the graph at the right, you can see that f^{-1} has domain $(-\infty, 1) \cup (1, \infty)$ and range $(-\infty, -2) \cup (-2, \infty)$. The domain and range of f are equal to the range and domain of f^{-1}, respectively. So, $f^{-1}(x) = \dfrac{-2x-1}{x-1}$ for $x \neq 1$.



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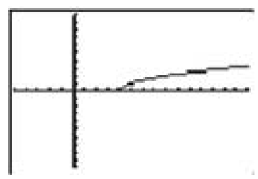
b. $f(x) = \sqrt{x-4}$

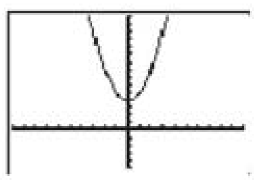
The graph of f shown passes the horizontal line test. Therefore, f is a one-to-one function and has an inverse function. From the graph, you can see that f has domain $[4, \infty)$ and range $[0, \infty)$. Now find f^{-1}.



$[-5, 15]$ scl: 1 by $[-10, 10]$ scl: 1

$f(x) = \sqrt{x-4}$	Original function
$y = \sqrt{x-4}$	Replace $f(x)$ with y.
$x = \sqrt{y-4}$	Interchange x and y.
$x^2 = y - 4$	Square each side.
$y = x^2 + 4$	Solve for y.
$f^{-1}(x) = x^2 + 4$	Replace y with $f^{-1}(x)$.

From the graph of $y = x^2 + 4$ shown, you can see that the inverse relation has domain $(-\infty, \infty)$ and range $[4, \infty)$. By restricting the domain of the inverse relation to $[0, \infty)$, the domain and range of f are equal to the range and domain of f^{-1}, respectively. So, $f^{-1}(x) = x^2 + 4$, for $x \geq 0$.



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GuidedPractice

2A. $f(x) = -16 + x^3$ **2B.** $f(x) = \dfrac{x+7}{x}$ **2C.** $f(x) = \sqrt{x^2 - 20}$

An inverse function f^{-1} has the effect of "undoing" the action of a function f. For this reason, inverse functions can also be defined in terms of their composition with each other.

KeyConcept Compositions of Inverse Functions

Two functions, f and g, are inverse functions if and only if

- $f[g(x)] = x$ for every x in the domain of $g(x)$ and
- $g[f(x)] = x$ for every x in the domain of $f(x)$.

StudyTip

Inverse Functions The biconditional statement "if and only if" in the definition of inverse functions means that if g is the inverse of f, then it is also true that f is the inverse of g.

Notice that the composition of a function with its inverse function is always the identity function. You can use this fact to verify that two functions are inverse functions of each other.

Example 3 Verify Inverse Functions

Show that $f(x) = \frac{6}{x-4}$ and $g(x) = \frac{6}{x} + 4$ are inverse functions.

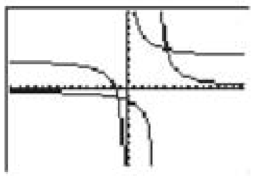
Show that $f[g(x)] = x$ and that $g[f(x)] = x$.

$$f[g(x)] = f\left(\frac{6}{x} + 4\right) \qquad\qquad g[f(x)] = g\left(\frac{6}{x-4} + 4\right)$$

$$= \frac{6}{\left(\frac{6}{x} + 4\right) - 4} \qquad\qquad = \frac{6}{\left(\frac{6}{x-4}\right)} + 4$$

$$= \frac{6}{\left(\frac{6}{x}\right)} \text{ or } x \qquad\qquad = x - 4 + 4 \text{ or } x$$

Because $f[g(x)] = g[f(x)] = x$, $f(x)$ and $g(x)$ are inverse functions. This is supported graphically because $f(x)$ and $g(x)$ appear to be reflections of each other in the line $y = x$.



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GuidedPractice

Show that f and g are inverse functions.

3A. $f(x) = 18 - 3x$, $g(x) = 6 - \frac{x}{3}$

3B. $f(x) = x^2 + 10$, $x \geq 0$; $g(x) = \sqrt{x - 10}$

The inverse functions of most one-to-one functions are often difficult to find algebraically. However, it is possible to graph the inverse function by reflecting the graph of the original function in the line $y = x$.

Example 4 Find Inverse Functions Graphically

Use the graph of $f(x)$ in Figure 1.7.3 to graph $f^{-1}(x)$.

Graph the line $y = x$. Locate a few points on the graph of $f(x)$. Reflect these points in $y = x$. Then connect them with a smooth curve that mirrors the curvature of $f(x)$ in line $y = x$ (Figure 1.7.4).

StudyTip

Inverse Functions and Extrema A continuous function has an inverse function if and only if it has no local maxima or minima. If the function does have a local maximum or minimum, then it will not pass the horizontal line test, and is not a one-to-one function.

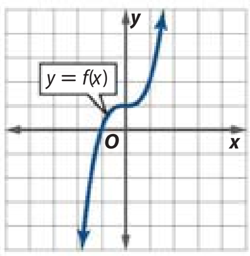


Figure 1.7.3

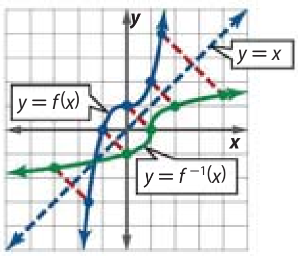
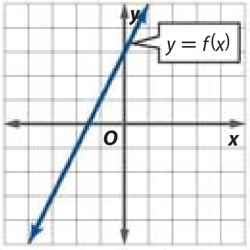


Figure 1.7.4

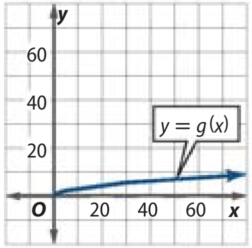
GuidedPractice

Use the graph of each function to graph its inverse function.

4A.



4B.



Real-World Example 5 Use an Inverse Function

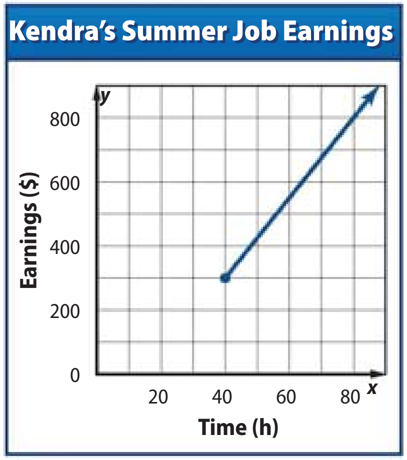
SUMMER EARNINGS **Kendra earns \$8 an hour, works at least 40 hours per week, and receives overtime pay at 1.5 times her regular hourly rate for any time over 40 hours. Her total earnings $f(x)$ for a week in which she worked x hours is given by $f(x) = 320 + 12(x - 40)$.**

a. **Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.**

The function simplifies to $f(x) = 320 + 12x - 480$ or $12x - 160$. The graph of $f(x)$ passes the horizontal line test. Therefore, $f(x)$ is a one-to-one function and has an inverse function.

Find $f^{-1}(x)$.

$f(x) = 12x - 160$	Original function
$y = 12x - 160$	Replace $f(x)$ with y.
$x = 12y - 160$	Interchange x and y.
$x + 160 = 12y$	Add 160 to each side.
$y = \dfrac{x + 160}{12}$	Solve for y.
$f^{-1}(x) = \dfrac{x + 160}{12}$	Replace y with $f^{-1}(x)$.



Real-WorldLink

From 1999 to 2006, the number of 16- to 19-year-olds in the United States who had summer jobs decreased from 48% to 37%.

Source: *U.S. Bureau of Labor Statistics*

b. **What do $f^{-1}(x)$ and x represent in the inverse function?**

In the inverse function, x represents Kendra's earnings for a particular week and $f^{-1}(x)$ represents the number of hours Kendra worked that week.

c. **What restrictions, if any, should be placed on the domain of $f(x)$ and $f^{-1}(x)$? Explain.**

The function $f(x)$ assumes that Kendra works at least 40 hours in a week. There are $7 \cdot 24$ or 168 hours in a week, so the domain of $f(x)$ is [40, 168]. Because $f(40) = 320$ and $f(168) = 1856$, the range of $f(x)$ is [320, 1856]. Because the range of $f(x)$ must equal the domain of $f^{-1}(x)$, the domain of $f^{-1}(x)$ is [320, 1856].

d. **Find the number of hours Kendra worked last week if her earnings were \$380.**

Because $f^{-1}(380) = \dfrac{380 + 160}{12}$ or 45, Kendra worked 45 hours last week.

GuidedPractice

5. **SAVINGS** Solada's net pay is 65% of her gross pay, and she budgets \$600 per month for living expenses. She estimates that she can save 20% of her remaining money, so her one-month savings $f(x)$ for a gross pay of x dollars is given by $f(x) = 0.2(0.65x - 600)$.

A. Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.

B. What do $f^{-1}(x)$ and x represent in the inverse function?

C. What restrictions, if any, should be placed on the domains of $f(x)$ and $f^{-1}(x)$? Explain.

D. Determine Solada's gross pay for one month if her savings for that month were \$120.

Exercises

= Step-by-Step Solutions begin on page R29.

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*. (Example 1)

1. $f(x) = x^2 + 6x + 9$

2. $f(x) = x^2 - 16x + 64$

3. $f(x) = x^2 - 10x + 25$

4. $f(x) = 3x - 8$

5. $f(x) = \sqrt{2x}$

6. $f(x) = 4$

7. $f(x) = \sqrt{x + 4}$

8. $f(x) = -4x^2 + 8$

9. $f(x) = \dfrac{5}{x - 6}$

10. $f(x) = \dfrac{8}{x + 2}$

11. $f(x) = x^3 - 9$

12. $f(x) = \frac{1}{4}x^3$

Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain. (Example 2)

13. $g(x) = -3x^4 + 6x^2 - x$

14. $f(x) = 4x^5 - 8x^4$

15. $h(x) = x^7 + 2x^3 - 10x^2$

16. $f(x) = \sqrt{x + 8}$

17. $f(x) = \sqrt{6 - x^2}$

18. $f(x) = |x - 6|$

19. $f(x) = \dfrac{4 - x}{x}$

20. $g(x) = \dfrac{x - 6}{x}$

21. $f(x) = \dfrac{6}{\sqrt{8 - x}}$

22. $g(x) = \dfrac{7}{\sqrt{x + 3}}$

23. $f(x) = \dfrac{6x + 3}{x - 8}$

24. $h(x) = \dfrac{x + 4}{3x - 5}$

25. $g(x) = |x + 1| + |x - 4|$

26. **SPEED** The speed of an object in kilometers per hour y is $y = 1.6x$, where x is the speed of the object in miles per hour. (Example 2)

a. Find an equation for the inverse of the function. What does each variable represent?

b. Graph each equation on the same coordinate plane.

Show algebraically that *f* and *g* are inverse functions. (Example 3)

27. $f(x) = -6x + 3$
$g(x) = \dfrac{3 - x}{6}$

28. $f(x) = 4x + 9$
$g(x) = \dfrac{x - 9}{4}$

29. $f(x) = -3x^2 + 5, x \geq 0$
$g(x) = \sqrt{\dfrac{5 - x}{3}}$

30. $f(x) = \dfrac{x^2}{4} + 8, x \geq 0$
$g(x) = \sqrt{4x - 32}$

31. $f(x) = 2x^3 - 6$
$g(x) = \sqrt[3]{\dfrac{x + 6}{2}}$

32. $f(x) = (x + 8)^{\frac{3}{2}}$
$g(x) = x^{\frac{2}{3}} - 8, x \geq 0$

33. $g(x) = \sqrt{x + 8} - 4$
$f(x) = x^2 + 8x + 8, x \geq -4$

34. $g(x) = \sqrt{x - 8} + 5$
$f(x) = x^2 - 10x + 33, x \geq 5$

35. $f(x) = \dfrac{x + 4}{x}$
$g(x) = \dfrac{4}{x - 1}$

36. $f(x) = \dfrac{x - 6}{x + 2}$
$g(x) = \dfrac{2x + 6}{1 - x}$

37. **PHYSICS** The kinetic energy of an object in motion in joules can be described by $f(x) = 0.5mx^2$, where m is the mass of the object in kilograms and x is the speed of the object in meters per second. (Example 3)

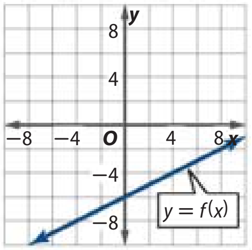
a. Find the inverse of the function. What does each variable represent?

b. Show that $f(x)$ and the function you found in part **a** are inverses.

c. Graph $f(x)$ and $f^{-1}(x)$ on the same graphing calculator screen if the mass of the object is 1 kilogram.

Use the graph of each function to graph its inverse function. (Example 4)

38.



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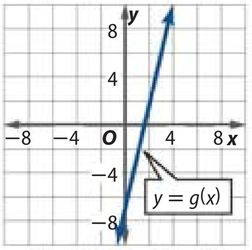
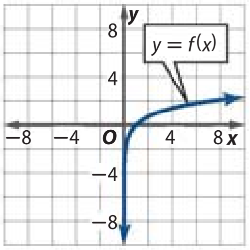


40.



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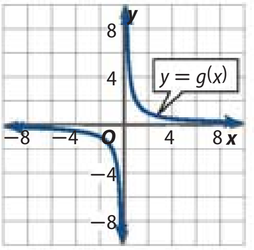

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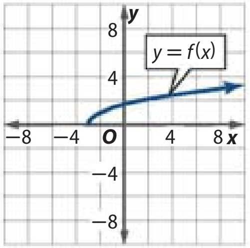

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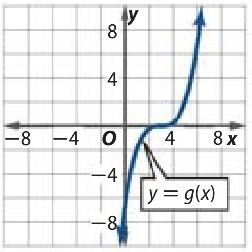


44. **JOBS** Jamie sells shoes at a department store after school. Her base salary each week is \$140, and she earns a 10% commission on each pair of shoes that she sells. Her total earnings $f(x)$ for a week in which she sold x dollars worth of shoes is $f(x) = 140 + 0.1x$. (Example 5)

a. Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.

b. What do $f^{-1}(x)$ and x represent in the inverse function?

c. What restrictions, if any, should be placed on the domains of $f(x)$ and $f^{-1}(x)$? Explain.

d. Find Jamie's total sales last week if her earnings for that week were \$220.

45. **CURRENCY** The average exchange rate from Euros to U.S. dollars for a recent four-month period can be described by $f(x) = 0.66x$, where x is the currency value in Euros. (Example 5)

a. Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.

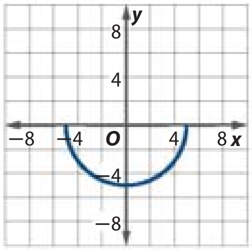
b. What do $f^{-1}(x)$ and x represent in the inverse function?

c. What restrictions, if any, should be placed on the domains of $f(x)$ and $f^{-1}(x)$? Explain.

d. What is the value in Euros of 100 U.S. dollars?

Determine whether each function has an inverse function.

46.



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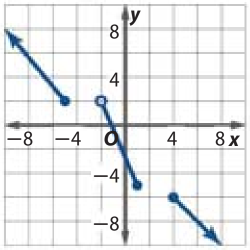

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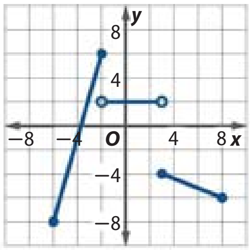

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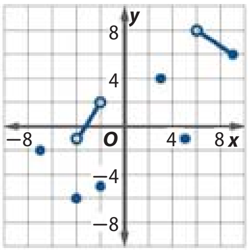


Determine if f^{-1} exists. If so, complete a table for f^{-1}.

50.

x	−6	−4	−1	3	6	10
$f(x)$	−4	0	3	5	9	13

51.

x	−3	−2	−1	0	1	2
$f(x)$	14	11	8	10	11	16

52.

x	1	2	3	4	5	6
$f(x)$	2	8	16	54	27	16

53.

x	−10	−9	−8	−7	−6	−5
$f(x)$	8	7	6	5	4	3

54. **TEMPERATURE** The formula $f(x) = \frac{9}{5}x + 32$ is used to convert x degrees Celsius to degrees Fahrenheit. To convert x degrees Fahrenheit to Kelvin, the formula $k(x) = \frac{5}{9}(x + 459.67)$ is used.

a. Find f^{-1}. What does this function represent?

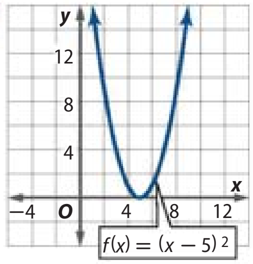
b. Show that f and f^{-1} are inverse functions. Graph each function on the same graphing calculator screen.

c. Find $[k \circ f](x)$. What does this function represent?

d. If the temperature is 60°C, what would the temperature be in Kelvin?

Restrict the domain of each function so that the resulting function is one-to-one. Then determine the inverse of the function.

55.



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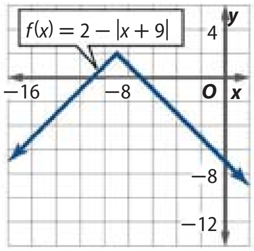

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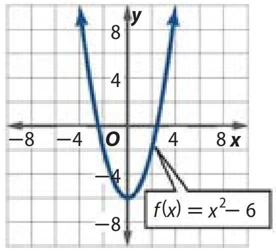

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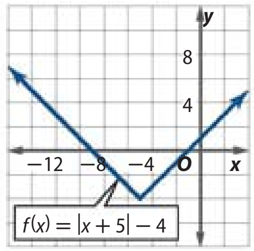


State the domain and range of f and f^{-1}, if f^{-1} exists.

59. $f(x) = \sqrt{x - 6}$

60. $f(x) = x^2 + 9$

61. $f(x) = \dfrac{3x + 1}{x - 4}$

62. $f(x) = \dfrac{8x + 3}{2x - 6}$

63. **ENVIRONMENT** Once an endangered species, the bald eagle was downlisted to threatened status in 1995. The table shows the number of nesting pairs each year.

Year	Nesting Pairs
1984	1757
1990	3035
1994	4449
1998	5748
2000	6471
2005	7066

a. Use the table to approximate a linear function that relates the number of nesting pairs to the year. Let 0 represent 1984.

b. Find the inverse of the function you generated in part **a.** What does each variable represent?

c. Using the inverse function, in approximately what year was the number of nesting pairs 5094?

64. **FLOWERS** Bonny needs to purchase 75 flower stems for banquet decorations. She can choose between lilies and hydrangea, which cost \$5.00 per stem and \$3.50 per stem, respectively.

a. Write a function for the total cost of the flowers.

b. Find the inverse of the cost function. What does each variable represent?

c. Find the domain of the cost function and its inverse.

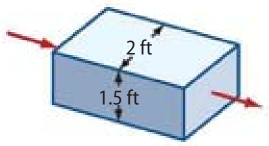
d. If the total cost for the flowers was \$307.50, how many lilies did Bonny purchase?

Find an equation for the inverse of each function, if it exists. Then graph the equations on the same coordinate plane. Include any domain restrictions.

65. $f(x) = \begin{cases} x^2 & \text{if } -4 \geq x \\ -2x + 5 & \text{if } -4 < x \end{cases}$

66. $f(x) = \begin{cases} -4x + 6 & \text{if } -5 \geq x \\ 2x - 8 & \text{if } -5 < x \end{cases}$

67. FLOW RATE The flow rate of a gas is the volume of gas that passes through an area during a given period of time. The speed v of air flowing through a vent can be found using $v(r) = \frac{r}{A}$, where r is the flow rate in cubic feet per second and A is the cross-sectional area of the vent in square feet.



a. Find v^{-1} of the vent shown. What does this function represent?

b. Determine the speed of air flowing through the vent in feet per second if the flow rate is 15,000 feet cubed per second.

c. Determine the gas flow rate of a circular vent that has a diameter of 5 feet with a gas stream that is moving at 1.8 feet per second.

68. COMMUNICATION A cellular phone company is having a sale as shown. Assume that the $50 rebate is given only after the 10% discount is given.

a. Write a function r for the price of the phone as a function of the original price if only the rebate applies.

b. Write a function d for the price of the phone as a function of the original price if only the discount applies.

c. Find a formula for $T(x) = [r \circ d](x)$ if both the discount and the rebate apply.

d. Find T^{-1} and explain what the inverse represents.

e. If the total cost of the phone after the discount and the rebate was $49, what was the original price of the phone?

Use $f(x) = 8x - 4$ and $g(x) = 2x + 6$ to find each of the following.

69. $[f^{-1} \circ g^{-1}](x)$

70. $[g^{-1} \circ f^{-1}](x)$

71. $[f \circ g]^{-1}(x)$

72. $[g \circ f]^{-1}(x)$

73. $(f \cdot g)^{-1}(x)$

74. $(f^{-1} \cdot g^{-1})(x)$

Use $f(x) = x^2 + 1$ with domain $[0, \infty)$ and $g(x) = \sqrt{x - 4}$ to find each of the following.

75. $[f^{-1} \circ g^{-1}](x)$

76. $[g^{-1} \circ f^{-1}](x)$

77. $[f \circ g]^{-1}(x)$

78. $[g \circ f]^{-1}(x)$

79. $(f \cdot g^{-1})(x)$

80. $(f^{-1} \cdot g)(x)$

81. COPIES Karen's Copies charges users $0.40 for every minute or part of a minute to use their computer scanner. Suppose you use the scanner for x minutes, where x is any real number greater than 0.

a. Sketch the graph of the function, $C(x)$, that gives the cost of using the scanner for x minutes.

b. What are the domain and range of $C(x)$?

c. Sketch the graph of the inverse of $C(x)$.

d. What are the domain and range of the inverse?

e. What real-world situation is modeled by the inverse?

82. MULTIPLE REPRESENTATIONS In this problem, you will investigate inverses of even and odd functions.

a. GRAPHICAL Sketch the graphs of three different even functions. Do the graphs pass the horizontal line test?

b. ANALYTICAL What pattern can you discern regarding the inverses of even functions? Confirm or deny the pattern algebraically.

c. GRAPHICAL Sketch the graphs of three different odd functions. Do the graphs pass the horizontal line test?

d. ANALYTICAL What pattern can you discern regarding the inverses of odd functions? Confirm or deny the pattern algebraically.

H.O.T. Problems Use Higher-Order Thinking Skills

83. REASONING If f has an inverse and a zero at 6, what can you determine about the graph of f^{-1}?

84. WRITING IN MATH Explain what type of restriction on the domain is needed to determine the inverse of a quadratic function and why a restriction is needed. Provide an example.

85. REASONING *True* or *False*. Explain your reasoning. *All linear functions have inverse functions.*

86. CHALLENGE If $f(x) = x^3 - ax + 8$ and $f^{-1}(23) = 3$, find the value of a.

87. REASONING Can $f(x)$ pass the horizontal line test when $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = 0$? Explain.

88. REASONING Why is $\pm$ not used when finding the inverse function of $f(x) = \sqrt{x + 4}$?

89. WRITING IN MATH Explain how an inverse of f can exist. Give an example provided that the domain of f is restricted and f does not have an inverse when the domain is unrestricted.

Spiral Review

For each pair of functions, find $f \circ g$ and $g \circ f$. Then state the domain of each composite function. (Lesson 1-6)

90. $f(x) = x^2 - 9$
$g(x) = x + 4$

91. $f(x) = \frac{1}{2}x - 7$
$g(x) = x + 6$

92. $f(x) = x - 4$
$g(x) = 3x^2$

Use the graph of the given parent function to describe the graph of each related function. (Lesson 1-5)

93. $f(x) = x^2$
- **a.** $g(x) = (0.2x)^2$
- **b.** $h(x) = (x - 5)^2 - 2$
- **c.** $m(x) = 3x^2 + 6$

94. $f(x) = x^3$
- **a.** $g(x) = |x^3 + 3|$
- **b.** $h(x) = -(2x)^3$
- **c.** $m(x) = 0.75(x + 1)^3$

95. $f(x) = |x|$
- **a.** $g(x) = |2x|$
- **b.** $h(x) = |x - 5|$
- **c.** $m(x) = |3x| - 4$

96. ADVERTISING A newspaper surveyed companies on the annual amount of money spent on television commercials and the estimated number of people who remember seeing those commercials each week. A soft-drink manufacturer spends \$40.1 million a year and estimates 78.6 million people remember the commercials. For a package-delivery service, the budget is \$22.9 million for 21.9 million people. A telecommunications company reaches 88.9 million people by spending \$154.9 million. Use a matrix to represent these data. (Lesson 0-6)

Solve each system of equations. (Lesson 0-5)

97. $x + 2y + 3z = 5$
$3x + 2y - 2z = -13$
$5x + 3y - z = -11$

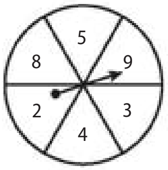
98. $7x + 5y + z = 0$
$-x + 3y + 2z = 16$
$x - 6y - z = -18$

99. $x - 3z = 7$
$2x + y - 2z = 11$
$-x - 2y + 9z = 13$

100. BASEBALL A batter pops up the ball. Suppose the ball was 3.5 feet above the ground when he hit it straight up with an initial velocity of 80 feet per second. The function $d(t) = 80t - 16t^2 + 3.5$ gives the ball's height above the ground in feet as a function of time t in seconds. How long did the catcher have to get into position to catch the ball after it was hit? (Lesson 0-3)

Skills Review for Standardized Tests

101. SAT/ACT What is the probability that the spinner will land on a number that is either even or greater than 5?



A $\frac{1}{6}$ C $\frac{1}{2}$ E $\frac{5}{6}$

B $\frac{1}{3}$ D $\frac{2}{3}$

102. REVIEW If m and n are both odd natural numbers, which of the following must be true?

I. $m^2 + n^2$ is even.
II. $m^2 + n^2$ is divisible by 4.
III. $(m + n)^2$ is divisible by 4.

F none H I and II only
G I only J I and III only

103. Which of the following is the inverse of $f(x) = \frac{3x - 5}{2}$?

A $g(x) = \frac{2x + 5}{3}$
B $g(x) = \frac{3x + 5}{2}$
C $g(x) = 2x + 5$
D $g(x) = \frac{2x - 5}{3}$

104. REVIEW A train travels d miles in t hours and arrives at its destination 3 hours late. At what average speed, in miles per hour, should the train have gone in order to have arrived on time?

F $t - 3$
G $\frac{t - 3}{d}$
H $\frac{d}{t - 3}$
J $\frac{d}{t} - 3$

EXTEND 1-7

Graphing Technology Lab

Graphing Inverses using Parametric Equations

Objective

- Use a graphing calculator and parametric equations to graph inverses on the calculator.

NewVocabulary

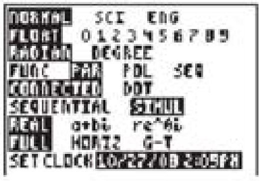
parametric equations

Parametric equations are equations that can express the position of an object as a function of time. The basic premise of parametric equations is the introduction of an extra variable t, called a *parameter*. For example, $y = x + 4$ can be expressed parametrically using $x = t$ and $y = t + 4$. Parametric equations will be studied in depth in Lesson 7-5.

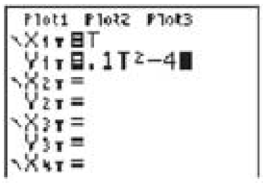
Activity 1 Parametric Graph

Graph $x = t$, $y = 0.1t^2 - 4$.

Step 1 Set the mode. In the MODE menu, select par and simul. This allows the equations to be graphed simultaneously.

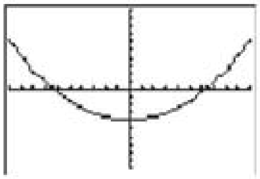


Step 2 Enter the parametric equations. In parametric form, X,T,θ,n will use t instead of x.



Step 3 Set the window as shown.

Step 4 Graph the equations. Notice that the graph looks like $y = 0.1x^2 - 4$ but is traced from $t = -10$ to $t = 10$.



[−10, 10] scl: 1 by [−10, 10] scl: 1
t: [−10, 10]; tstep: 1

StudyTip

Standard Window You can use ZoomStandard to set the window in standard form.

Exercises

1. **REASONING** Graph the equations using Tstep = 10, 5, 0.5, and 0.1. How does this affect the way the graph is shown?

2. In this problem, you will investigate the relationship between x, y, and t.

   a. Graph $X_{1T} = t - 3$, $Y_{1T} = t + 4$ in the standard viewing window.

   b. Replace the equations in part **a** with $X_{1T} = t$, $Y_{1T} = t + 7$ and graph.

   c. What do you notice about the two graphs?

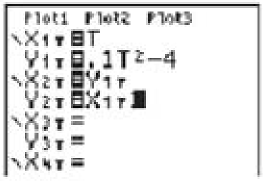
   d. **REASONING** What conclusions can you make about the relationship between x, y, and t? In other words, how do you think the second set of parametric equations was formed using the first set?

One benefit of parametric equations is the ability to graph inverses without determining them.

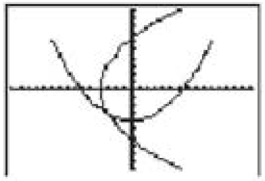
Activity 2 Graph an Inverse

Graph the inverse of $x = t$, $y = 0.1t^2 - 4$.

Step 1 Enter the given equations as X_{1T} and Y_{1T}. To graph the inverse, set $X_{2T} = Y_{1T}$ and $Y_{2T} = X_{1T}$. These are found in the VARS menu. Select Y-Vars, parametric, X_{1T}.



Step 2 Graph the relation and the inverse. You can use ZSquare to see the symmetry of the two graphs more clearly.



[−15, 15] scl: 1 by [−10, 10] scl: 1
t: [−10, 10]; $t$step: 1

Exercises

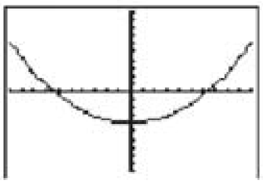
3. **REASONING** What needs to be true about the ordered pairs of each graph in Activity 2?

4. **REASONING** Does the graph of $x = t$, $y = 0.1t^2 - 4$ represent a one-to-one function? Explain.

Activity 3 Domains and One-to-One Functions

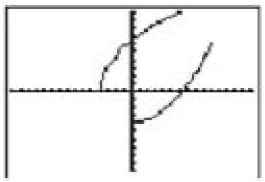
Limit the domain of $x = t$, $y = 0.1t^2 - 4$ in order to make it one-to-one.

Step 1 The minimum of the graph is located at $t = 0$. We can produce a one-to-one function for t values $0 \leq t \leq 10$.



[−10, 10] scl: 1 by [−10, 10] scl: 1
t: [−10, 10]; $t$step: 1

Step 2 Change Tmin from −10 to 0 in order to limit the domain. Graph the one-to-one function and its inverse.



[−15, 15] scl: 1 by [−10, 10] scl: 1
t: [0, 10]; $t$step: 1

StudyTip

Symmetry You may need to use the TRACE feature and adjust Tstep in order to locate the axis of symmetry.

Exercises

Graph each function. Then graph the inverse function and indicate the limited domain if necessary.

5. $x = t - 6$, $y = t^2 + 2$

6. $x = 3t - 1$, $y = t^2 + t$

7. $x = 3 - 2t$, $y = t^2 - 2t + 1$

8. $x = 2t^2 + 3$, $y = \sqrt{t}$

9. $x = 4t$, $y = \sqrt{t + 2}$

10. $x = t - 8$, $y = t^3$

11. **CHALLENGE** Consider a quintic function with two relative maxima and two relative minima. Into how many different one-to-one functions can this function be separated if each separation uses the largest interval possible?

CHAPTER 1

Study Guide and Review

Study Guide

KeyConcepts

Functions (Lesson 1-1)

- Common subsets of the real numbers are integers, rational numbers, irrational numbers, whole numbers, and natural numbers.
- A function *f* is a relation that assigns each element in the domain exactly one element in the range.
- The graph of a function passes the vertical line test.

Analyzing Graphs of Functions and Relations (Lesson 1-2)

- Graphs may be symmetric with respect to the *y*-axis, the *x*-axis, and the origin.
- An even function is symmetric with respect to the *y*-axis. An odd function is symmetric with respect to the origin.

Continuity, End Behavior, and Limits (Lesson 1-3)

- If the value of $f(x)$ approaches a unique value L as x approaches c from either side, then the limit of $f(x)$ as x approaches c is L. It is written $\lim_{x \to c} f(x) = L$.
- A function may be discontinuous because of infinite discontinuity, jump discontinuity, or removable discontinuity.

Extrema and Average Rate of Change (Lesson 1-4)

- A function can be described as increasing, decreasing, or constant.
- Extrema of a function include relative maxima and minima and absolute maxima and minima.
- The average rate of change between two points can be represented by $m_{sec} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Parent Functions and Transformations (Lesson 1-5)

Transformations of parent functions include translations, reflections, and dilations.

Operations with and Composition of Functions (Lesson 1-6)

The sum, difference, product, quotient, and composition of two functions form new functions.

Inverse Relations and Functions (Lesson 1-7)

- Two relations are inverse relations if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b).
- Two functions, f and f^{-1}, are inverse functions if and only if $f[f^{-1}(x)] = x$ and $f^{-1}[f(x)] = x$.

KeyVocabulary

composition (p. 58)
constant (p. 34)
continuous function (p.24)
decreasing function (p. 34)
dilation (p. 49)
discontinuous function (p. 24)
end behavior (p. 28)
even function (p. 18)
extrema (p. 36)
function (p. 5)
increasing (p. 34)
interval notation (p. 5)
inverse function (p. 65)
inverse relation (p. 65)
limit (p. 24)
line symmetry (p. 16)
maximum (p. 36)
minimum (p. 36)
nonremovable discontinuity (p. 25)
odd function (p. 18)
one-to-one (p. 66)
parent function (p. 45)
piecewise-defined function (p. 8)
point symmetry (p. 16)
reflection (p. 48)
roots (p. 15)
translation (p. 47)
zero function (p. 45)
zeros (p. 15)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A <u>function</u> assigns every element of its domain to exactly one element of its range.
2. Graphs that have <u>point symmetry</u> can be rotated 180° with respect to a point and appear unchanged.
3. An <u>odd function</u> has a point of symmetry.
4. The graph of a <u>continuous function</u> has no breaks or holes.
5. The <u>limit</u> of a graph describes approaching a value without necessarily ever reaching it.
6. A function $f(x)$ with values that decrease as x increases is a <u>decreasing</u> function.
7. The <u>extrema</u> of a function can include relative maxima or minima.
8. The <u>translation</u> of a graph produces a mirror image of the graph with respect to a line.
9. A <u>one-to-one function</u> passes the horizontal line test.
10. One-to-one functions have <u>line symmetry</u>.

Lesson-by-Lesson Review

1-1 Functions (pp. 4–12)

Determine whether each relation represents y as a function of x.

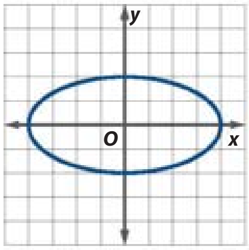
11. $3x - 2y = 18$

12. $y^3 - x = 4$

13.

x	y
5	7
7	9
9	11
11	13

14.



Let $f(x) = x^2 - 3x + 4$. Find each function value.

15. $f(5)$

16. $f(-3x))$

State the domain of each function.

17. $f(x) = 5x^2 - 17x + 1$

18. $g(x) = \sqrt{6x - 3}$

19. $h(a) = \dfrac{5}{a + 5}$

20. $v(x) = \dfrac{x}{x^2 - 4}$

Example 1

Determine whether $y^2 - 8 = x$ represents y as a function of x.

Solve for y.

$y^2 - 8 = x$	Original equation
$y^2 = x + 8$	Add 8 to each side.
$y = \pm\sqrt{x + 8}$	Take the square root of each side.

This equation does not represent y as a function of x because for any x-value greater than -8, there will be two corresponding y-values.

Example 2

Let $g(x) = -3x^2 + x - 6$. Find $g(2)$.

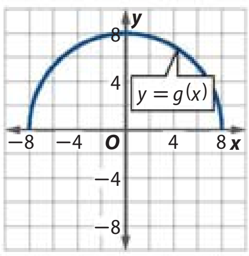
Substitute 2 for x in the expression $-3x^2 + x - 6$.

$g(2) = -3(2)^2 + 2 - 6$	$x = 2$
$= -12 + 2 - 6$ or -16	Simplify.

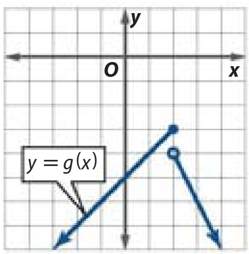
1-2 Analyzing Graphs of Functions and Relations (pp. 13–23)

Use the graph of g to find the domain and range of each function.

21.



22.



Find the y-intercept(s) and zeros for each function.

23. $f(x) = 4x - 9$

24. $f(x) = x^2 - 6x - 27$

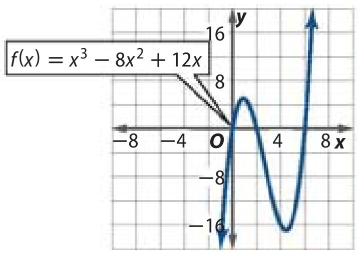
25. $f(x) = x^3 - 16x$

26. $f(x) = \sqrt{x + 2} - 1$

Example 3

Use the graph of $f(x) = x^3 - 8x^2 + 12x$ to find its y-intercept and zeros. Then find these values algebraically.

Estimate Graphically



It appears that $f(x)$ intersects the y-axis at $(0, 0)$, so the y-intercept is 0.

The x-intercepts appear to be at about 0, 2, and 6.

Solve Algebraically

Find $f(0)$.

$f(0) = (0)^3 - 8(0)^2 + 12(0)$ or 0

The y-intercept is 0.

Factor the related equation.

$x(x^2 - 8x + 12) = 0$

$x(x - 6)(x - 2) = 0$

The zeros of f are 0, 6, and 2.

Study Guide and Review *Continued*

1-3 Continuity, End Behavior, and Limits (pp. 24–33)

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as infinite, jump, or removable.

27. $f(x) = x^2 - 3x$; $x = 4$.

28. $f(x) = \sqrt{2x - 4}$; $x = 10$

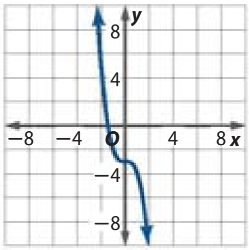
29. $f(x) = \frac{x}{x+7}$; $x = 0$ and $x = 7$

30. $f(x) = \frac{x}{x^2 - 4}$; $x = 2$ and $x = 4$

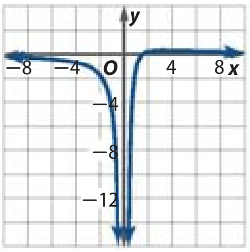
31. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$; $x = 1$

Use the graph of each function to describe its end behavior.

32.



33.



Example 4

Determine whether $f(x) = \frac{1}{x-4}$ is continuous at $x = 0$ and $x = 4$. Justify your answer using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

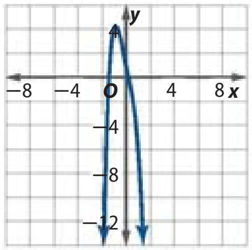
$f(0) = -0.25$, so f is defined at 0. The function values suggest that as f gets closer to -0.25 x gets closer to 0.

x	−0.1	−0.01	0	0.01	0.1
$f(x)$	−0.244	−0.249	−0.25	−0.251	−0.256

Because $\lim_{x \to 0} f(x)$ is estimated to be -0.25 and $f(0) = -0.25$, we can conclude that $f(x)$ is continuous at $x = 0$.
Because f is not defined at 4, f is not continuous at 4.

Example 5

Use the graph of $f(x) = -2x^4 - 5x + 1$ to describe its end behavior.



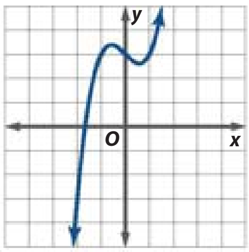
Examine the graph of $f(x)$.

As $x \to \infty$, $f(x) \to -\infty$.
As $x \to -\infty$, $f(x) \to -\infty$.

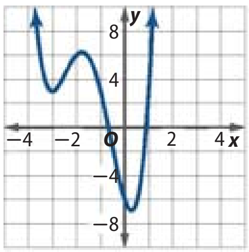
1-4 Extreme and Average Rate of Change (pp. 34–43)

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Then estimate to the nearest 0.5 unit, and classify the extrema for the graph of each function.

34.



35.



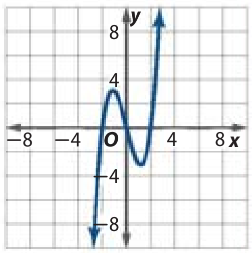
Find the average rate of change of each function on the given interval.

36. $f(x) = -x^3 + 3x + 1$; $[0, 2]$

37. $f(x) = x^2 + 2x + 5$; $[-5, 3]$

Example 6

Use the graph of $f(x) = x^3 - 4x$ to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Then estimate to the nearest 0.5 unit and classify the extrema for the graph of each function.



From the graph, we can estimate that f is increasing on $(-\infty, -1)$, decreasing on $(-1, 1)$, and increasing on $(1, \infty)$.

We can estimate that f has a relative maximum at $(-1, 3)$ and a relative minimum at $(1, -3)$.

1-5 Parent Functions and Transformations (pp. 45–55)

Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

38. $g(x) = \sqrt{x-3} + 2$

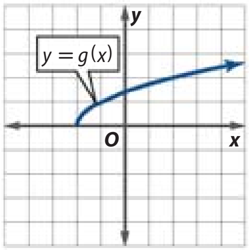
39. $g(x) = -(x-6)^2 - 5$

40. $g(x) = \dfrac{1}{2(x+7)}$

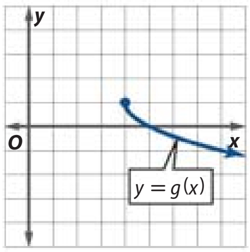
41. $g(x) = \dfrac{1}{4}[\![x]\!] + 3$

Describe how the graphs of $f(x) = \sqrt{x}$ and $g(x)$ are related. Then write an equation for $g(x)$.

42.



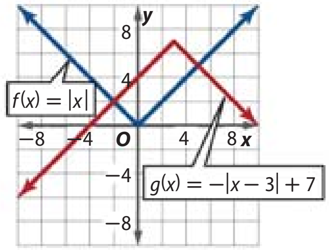
43.



Example 7

Identify the parent function $f(x)$ of $g(x) = -|x-3| + 7$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

The parent function for $g(x)$ is $f(x) = |x|$. The graph of g will be the same as the graph of f reflected in the x-axis, translated 3 units to the right, and translated 7 units up.



1-6 Function Operations and Composition of Functions (pp. 57–64)

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

44. $f(x) = x + 3$
$g(x) = 2x^2 + 4x - 6$

45. $f(x) = 4x^2 - 1$
$g(x) = 5x - 1$

46. $f(x) = x^3 - 2x^2 + 5$
$g(x) = 4x^2 - 3$

47. $f(x) = \dfrac{1}{x}$
$g(x) = \dfrac{1}{x^2}$

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](2)$.

48. $f(x) = 4x - 11$; $g(x) = 2x^2 - 8$

49. $f(x) = x^2 + 2x + 8$; $g(x) = x - 5$

50. $f(x) = x^2 - 3x + 4$; $g(x) = x^2$

Find $f \circ g$.

51. $f(x) = \dfrac{1}{x-3}$
$g(x) = 2x - 6$

52. $f(x) = \sqrt{x-2}$
$g(x) = 6x - 7$

Example 8

Given $f(x) = x^3 - 1$ and $g(x) = x + 7$, find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$. State the domain of each new function.

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x^3 - 1) + (x + 7)\\ &= x^3 + x + 6\end{aligned}$$

The domain of $(f+g)(x)$ is $(-\infty, \infty)$.

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x^3 - 1) - (x + 7)\\ &= x^3 - x - 8\end{aligned}$$

The domain of $(f-g)(x)$ is $(-\infty, \infty)$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x)\\ &= (x^3 - 1)(x + 7)\\ &= x^4 + 7x^3 - x - 7\end{aligned}$$

The domain of $(f \cdot g)(x)$ is $(-\infty, \infty)$.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ or } \frac{x^3 - 1}{x + 7}$$

The domain of $\left(\frac{f}{g}\right)(x)$ is $D = (-\infty, -7) \cup (-7, \infty)$.

1-7 Inverse Relations and Functions (pp. 65–73)

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

53. $f(x) = |x| + 6$

54. $f(x) = x^3$

55. $f(x) = -\frac{3}{x+6}$

56. $f(x) = x^3 - 4x^2$

Find the inverse function and state any restrictions on the domain.

57. $f(x) = x^3 - 2$

58. $g(x) = -4x + 8$

59. $h(x) = 2\sqrt{x+3}$

60. $f(x) = \frac{x}{x+2}$

Example 9

Find the inverse function of $f(x) = \sqrt{x} - 3$ and state any restrictions on its domain.

Note that f has domain $[0, \infty)$ and range $[-3, \infty)$. Now find the inverse relation of f.

$y = \sqrt{x} - 3$ Replace $f(x)$ with y.

$x = \sqrt{y} - 3$ Interchange x and y.

$x + 3 = \sqrt{y}$ Add 3 to each side.

$(x + 3)^2 = y$ Square each side. Note that $D = (-\infty, \infty)$ and $R = [0, \infty)$.

The domain of $y = (x + 3)^2$ does not equal the range of f unless restricted to $[-3, \infty)$. So, $f^{-1}(x) = (x + 3)^2$ for $x \geq -3$.

Applications and Problem Solving

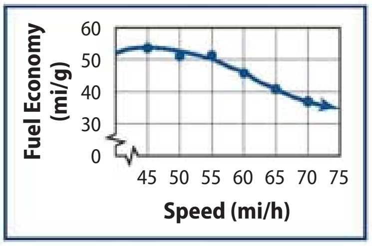
61. CELL PHONES Basic Mobile offers a cell phone plan that charges \$39.99 per month. Included in the plan are 500 daytime minutes that can be used Monday through Friday between 7 A.M. and 7 P.M. Users are charged \$0.20 per minute for every daytime minute over 500 used. (Lesson 1-1)

a. Write a function $p(x)$ for the cost of a month of service during which you use x daytime minutes.

b. How much will you be charged if you use 450 daytime minutes? 550 daytime minutes?

c. Graph $p(x)$.

62. AUTOMOBILES The fuel economy for a hybrid car at various highway speeds is shown. (Lesson 1-2)



a. Approximately what is the fuel economy for the car when traveling 50 miles per hour?

b. At approximately what speed will the car's fuel economy be less than 40 miles per gallon?

63. SALARIES After working for a company for five years, Ms. Washer was given a promotion. She is now earning \$1500 per month more than her previous salary. Will a function modeling her monthly income be a continuous function? Explain. (Lesson 1-3)

64. BASEBALL The table shows the number of home runs by a baseball player in each of the first 5 years he played professionally. (Lesson 1-4)

Year	2004	2005	2006	2007	2008
Number of Home Runs	5	36	23	42	42

a. Explain why 2006 represents a relative minimum.

b. Suppose the average rate of change of home runs between 2008 and 2011 is 5 home runs per year. How many home runs were there in 2011?

c. Suppose the average rate of change of home runs between 2007 and 2012 is negative. Compare the number of home runs in 2007 and 2012.

65. PHYSICS A stone is thrown horizontally from the top of a cliff. The velocity of the stone measured in meters per second after t seconds can be modeled by $v(t) = -\sqrt{(9.8t)^2 + 49}$. The speed of the stone is the absolute value of its velocity. Draw a graph of the stone's speed during the first 6 seconds. (Lesson 1-5)

66. FINANCIAL LITERACY A department store advertises \$10 off the price of any pair of jeans. How much will a pair of jeans cost if the original price is \$55 and there is 8.5% sales tax? (Lesson 1-6)

67. MEASUREMENT One inch is approximately equal to 2.54 centimeters. (Lesson 1-7)

a. Write a function $A(x)$ that will convert the area x of a rectangle from square inches to square centimeters.

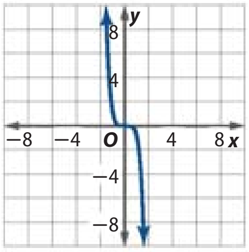
b. Write a function $A^{-1}(x)$ that will convert the area x of a rectangle from square centimeters to square inches.

CHAPTER 1

Practice Test

Determine whether the given relation represents y as a function of x.

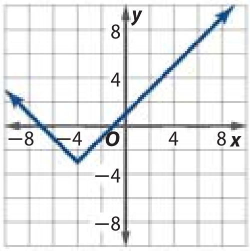
1. $x = y^2 - 5$

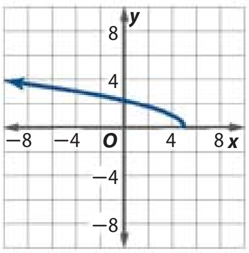
2. 

3. $y = \sqrt{x^2 + 3}$

4. **PARKING** The cost of parking a car downtown is \$0.75 per 30 minutes for a maximum of \$4.50. Parking is charged per second.
   a. Write a function for $c(x)$, the cost of parking a car for x hours.
   b. Find $c(2.5)$.
   c. What is the domain for $c(x)$? Explain your reasoning.

State the domain and range of each function.

5. 

6. 

Find the y-intercept(s) and zeros for each function.

7. $f(x) = 4x^2 - 8x - 12$

8. $f(x) = x^3 + 4x^2 + 3x$

9. **MULTIPLE CHOICE** Which relation is symmetric with respect to the x-axis?
   A $-x^2 - yx = 2$
   B $x^3y = 8$
   C $y = |x|$
   D $-y^2 = -4x$

Determine whether each function is continuous at $x = 3$. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

10. $f(x) = \begin{cases} 2x & \text{if } x < 3 \\ 9 - x & \text{if } x \geq 3 \end{cases}$

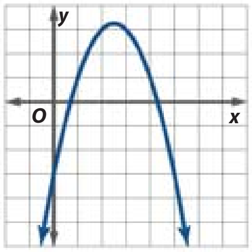
11. $f(x) = \dfrac{x - 3}{x^2 - 9}$

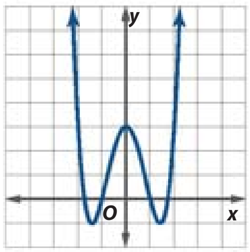
Find the average rate of change for each function on the interval $[-2, 6]$.

12. $f(x) = -x^4 + 3x$

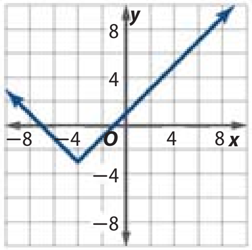
13. $f(x) = \sqrt{x + 3}$

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing or decreasing.

14. 

15. 

16. **MULTIPLE CHOICE** Which function is shown in the graph?



   F $f(x) = |x - 4| - 3$
   G $f(x) = |x - 4| + 3$
   H $f(x) = |x + 4| - 3$
   J $f(x) = |x + 4| + 3$

Identify the parent function $f(x)$ of $g(x)$. Then sketch the graph of $g(x)$.

17. $g(x) = -(x + 3)^3$

18. $g(x) = |x^2 - 4|$

Given $f(x) = x - 6$ and $g(x) = x^2 - 36$, find each function and its domain.

19. $\left(\frac{f}{g}\right)(x)$

20. $[g \circ f](x)$

21. **TEMPERATURE** In most countries, temperature is measured in degrees Celsius. The equation that relates degrees Fahrenheit with degrees Celsius is $F = \frac{9}{5}C + 32$.
   a. Write C as a function of F.
   b. Find two functions f and g such that $C = [f \circ g](F)$.

Determine whether f has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

22. $f(x) = (x - 2)^3$

23. $f(x) = \dfrac{x + 3}{x - 8}$

24. $f(x) = \sqrt{4 - x}$

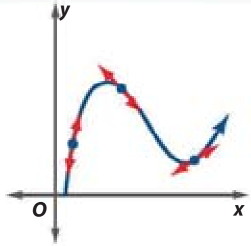
25. $f(x) = x^2 - 16$

Connect to AP Calculus
Rate of Change at a Point

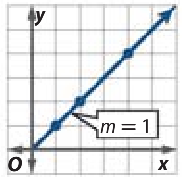
Objective

- Approximate the rate of change of a function at a point.

Differential calculus is a branch of calculus that focuses on the rates of change of functions at individual points. You have learned to calculate the constant rate of change, or slope, for linear functions and the average rate of change for nonlinear functions. Using differential calculus, you can determine the *exact* rate of change of any function at a single point, as represented by the slopes of the tangent lines in the figure at the right.



The constant rate of change for a linear function not only represents the slope of the graph between two points, but also the *exact* rate of change of the function at each of its points. For example, notice in the figure at the right that the slope m of the function is 1. This value also refers to the *exact* rate at which this function is changing at any point in its domain. This is the focus of differential calculus.



The average rate of change for nonlinear functions is represented by the slope of a line created by any two points on the graph of the function. This line is called a *secant line*. This slope is not the *exact* rate of change of the function at any one point. We can, however, use this process to give us an approximation for that instantaneous rate of change.

Activity 1 Approximate Rate of Change

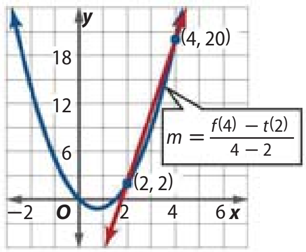
Approximate the rate of change of $f(x) = 2x^2 - 3x$ at $x = 2$.

Step 1 Graph $f(x) = 2x^2 - 3x$, and plot the point $P = (2, f(2))$.

Step 2 Draw a secant line through $P = (2, f(2))$ and $Q = (4, f(4))$.

Step 3 Calculate the average rate of change m for $f(x)$ using P and Q, as shown in the figure.

Step 4 Repeat Steps 1–3 four more times. Use $Q = (3, f(3))$, $Q = (2.5, f(2.5))$, $Q = (2.25, f(2.25))$, and $Q = (2.1, f(2.1))$.



Analyze the Results

1. As Q approaches P, what does the average rate of change m appear to approach?
2. Using a secant line to approximate rate of change at a point can produce varying results. Make a conjecture as to when this process will produce accurate approximations.

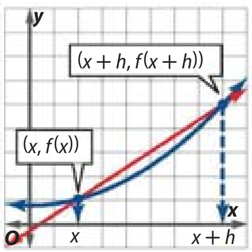
In differential calculus, we express the formula for average rate of change in terms of x and the horizontal distance h between the two points that determine the secant line.

Activity 2 Approximate Rate of Change

Write a general formula for finding the slope m of any secant line. Use the formula to approximate the rate of change of $f(x) = 2x^2 - 3x$ for $x = 2$.

Step 1 Generate an expression for finding the average rate of change for the figure at the right. This expression is called the *difference quotient*.

Step 2 Use your difference quotient to approximate the rate of change of $f(x)$ at $x = 2$. Let $h = 0.4, 0.25$, and 0.1.



Analyze the Results

3. As the value of h gets closer and closer to 0, what does the average rate of change appear to approach?
4. Graph $f(x)$ and the secant line created when $h = 0.1$.
5. What does the secant line appear to become as h approaches 0?
6. Make a conjecture about the rate of change of a function at a point as it relates to your answer to the previous question.

We can use the difference quotient to find the *exact* rate of change of a function at a single point.

Activity 3 Calculate Rate of Change

Use the difference quotient to calculate the *exact* rate of change of $f(x) = 2x^2 - 3x$ at the point $x = 2$.

Step 1 Substitute $x = 2$ into the difference quotient, as shown.

$$m = \frac{f(2 + h) - f(2)}{h}$$

Step 2 Expand the difference quotient by evaluating for $f(2 + h)$ and $f(2)$.

$$m = \frac{[2(2 + h)^2 - 3(2 + h)] - [2(2)^2 - 3(2)]}{h}$$

Step 3 Simplify the expression. At some point, you will need to factor h from the numerator and then reduce.

Step 4 Find the *exact* rate of change of $f(x)$ at $x = 2$ by substituting $h = 0$ into your expression.

Analyze the Results

7. Compare the *exact* rate of change found in Step 4 to the previous rates of change that you found.
8. What happens to the secant line for $f(x)$ at $x = 2$ when $h = 0$?
9. Explain the process for calculating the exact rate of change of a function at a point using the difference quotient.

Model and Apply

10. In this problem, you will approximate the rate of change for, and calculate the *exact* rate for, $f(x) = x^2 + 1$ at $x = 1$.

    a. Approximate the rate of change of $f(x)$ at $x = 1$ by calculating the average rates of change of the three secant lines through $f(3)$, $f(2)$, and $f(1.5)$. Graph $f(x)$ and the three secant lines on the same coordinate plane.

    b. Approximate the rate of change of $f(x)$ at $x = 1$ by using the difference quotient and three different values for h. Let $h = 0.4, 0.25$, and 0.1.

    c. Calculate the exact rate of change of $f(x)$ at $x = 1$ by first evaluating the difference quotient for $f(1 + h)$ and $f(1)$ and then substituting $h = 0$. Follow the steps in Activity 3.

CHAPTER 2

Power, Polynomial, and Rational Functions

Massimo Pizzotti/Photolibrary

Then

In **Chapter 1**, you analyzed functions and their graphs and determined whether inverse functions existed.

Now

In **Chapter 2**, you will:

- Model real-world data with polynomial functions.
- Use the Remainder and Factor Theorems.
- Find real and complex zeros of polynomial functions.
- Analyze and graph rational functions.
- Solve polynomial and rational inequalities.

Why?

ARCHITECTURE Polynomial functions are often used when designing and building a new structure. Architects use functions to determine the weight and strength of the materials, analyze costs, estimate deterioration of materials, and determine the proper labor force.

PREREAD Scan the lessons of Chapter 2, and use what you already know about functions to make a prediction of the purpose of this chapter.

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