CHAPTER 0

Preparing for Precalculus

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Now

Chapter 0 contains lessons on topics from previous courses. You can use this chapter in various ways.

- Begin the school year by taking the Pretest. If you need additional review, complete the lessons in this chapter. To verify that you have successfully reviewed the topics, take the Posttest.
- As you work through the text, you may find that there are topics you need to review. When this happens, complete the individual lessons that you need.
- Use this chapter for reference. When you have questions about any of these topics, flip back to this chapter to review definitions or key concepts.

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Get Started on the Chapter

You will review several new concepts, skills, and vocabulary terms as you study Chapter 0. To get ready, identify important terms and organize your resources.

Contents

ReviewVocabulary

English		Español
set	p. P3	conjunto
element	p. P3	elemento
subset	p. P3	subconjunto
universal set	p. P3	conjunto universal
complement	p. P3	complemento
union	p. P4	unión
intersection	p. P4	intersección
empty set	p. P4	conjunto vacío
imaginary unit	p. P6	unidad imaginario
complex number	p. P6	número complejo
standard form	p. P6	forma estándar
imaginary number	p. P6	número imaginario
complex conjugates	p. P7	conjugados complejos
parabola	p. P9	parábola
axis of symmetry	p. P9	eje de simetría
vertex	p. P9	vértice
completing the square	p. P12	completer el cuadrado
*n*th root	p. P14	raíz enésima
principal root	p. P14	raíz principal
system of equations	p. P18	sistema de ecuaciones
substitution method	p. P18	método de sustitución
elimination method	p. P19	método de eliminación
matrix	p. P23	matriz
element	p. P23	elemento
dimension	p. P23	dimensión
experiment	p. P28	experimento
factorial	p. P28	factorial
permutation	p. P29	permutación
combination	p. P30	combinación
measure of central tendency	p. P32	medida de tendencia central
measures of spread	p. P32	medidas de propagación
frequency distribution	p. P34	distribución de frecuencias

CHAPTER 0 Pretest

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*.

1. L is the set of whole number multiples of 2 that are less than 22. $18 \in L$

2. S is the set of integers that are less than 5 but greater than -6. $-8 \in S$

Let $C = \{0, 1, 2, 3, 4\}$, $D = \{3, 5, 7, 8\}$, $E = \{0, 1, 2\}$, and $F = \{0, 8\}$. Find each of the following.

3. $D \cap E$ **4.** $C \cap E$

5. $C \cap F$ **6.** $D \cup E$

Simplify.

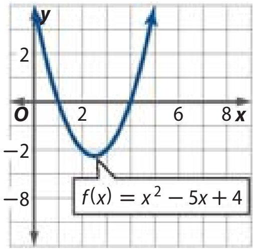
7. $(6 + 5\boldsymbol{i}) + (-3 + 2\boldsymbol{i})$ **8.** $(-3 + 4\boldsymbol{i}) - (4 - 5\boldsymbol{i})$

9. $(1 + 8\boldsymbol{i})(6 + 2\boldsymbol{i})$ **10.** $(-3 + 3\boldsymbol{i})(-2 + 2\boldsymbol{i})$

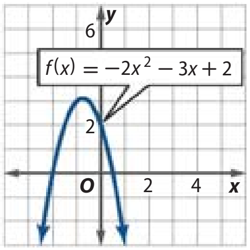
11. $\frac{3 + \boldsymbol{i}}{5 - 2\boldsymbol{i}}$ **12.** $\frac{-3 + \boldsymbol{i}}{4 - 3\boldsymbol{i}}$

Determine whether each function has a *maximum* or *minimum* value. Then find the value of the maximum or minimum, and state the domain and range of the function.

13.



14.



Solve each equation.

15. $x^2 - x - 20 = 0$ **16.** $x^2 - 3x + 5 = 0$

17. $x^2 + 2x - 1 = 0$ **18.** $x^2 + 11x + 24 = 0$

19. CARS The current value V and the original value v of a car are related by $V = v(1 - r)^n$, where r is the rate of depreciation per year and n is the number of years. If the original value of a car is \$10,000, what would be the current value of the car after 30 months at an annual depreciation rate of 10%?

Simplify each expression.

20. $\sqrt[6]{x^{12}y^{15}}$ **21.** $\sqrt[3]{8a^9b^7}$

22. $\sqrt{25r^5t^4u^2}$ **23.** $\sqrt[5]{32x^{11}y^{20}z^5}$

Simplify.

24. $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{4}}}$ **25.** $\sqrt[4]{81x^8y^{14}}$

26. $\sqrt[7]{x^{15}y^{23}}$ **27.** $\frac{\sqrt[8]{49}}{\sqrt[6]{7}}$

28. JOBS Destiny mows lawns for \$8 per lawn and weeds gardens for \$10 per garden. If she had 8 jobs and made \$72, how many of the jobs were mowing? How many were weeding?

Solve each system of equations. State whether the system is *consistent and independent, consistent and dependent*, or *inconsistent*.

29. $3x + y = 4$
$x - y = 12$

30. $2x - y = 2$
$-4x + 2y = -4$

31. $2x + 4y - z = -1$
$2x - 3y + 2z = 6$
$-x - 5y + z = -2$

32. $-3x + 9y - 3z = -12$
$-3x + y - z = -1$
$2x - 6y + 2z = 9$

Solve each system of inequalities. If the system has no solution, state *no solution*.

33. $y \geq x + 5$
$y \leq 2x + 2$

34. $y + x < 3$
$y > -2x - 4$

35. $4x - 3y < 7$
$2y - x < -6$

36. $3y \leq 2x - 8$
$y \geq \frac{2}{3}x - 1$

Find each of the following for $D = \begin{bmatrix} -2 & 4 \\ 0 & 1 \\ 4 & -3 \end{bmatrix}$, $E = \begin{bmatrix} 3 & 5 \\ -2 & -1 \\ 0 & -1 \end{bmatrix}$, and $F = \begin{bmatrix} 8 & 2 \\ -3 & -5 \\ 2 & 2 \end{bmatrix}$.

37. $D - F$ **38.** $D + 2F$ **39.** $2D - E$

40. $D + E + F$ **41.** $3D - 2E$ **42.** $D - 3E + 3F$

Find each permutation or combination.

43. ${}_9C_5$ **44.** ${}_9P_5$ **45.** ${}_5P_5$

46. ${}_5C_5$ **47.** ${}_4P_2$ **48.** ${}_4C_2$

49. CARDS Three cards are randomly drawn from a standard deck of 52 cards. Find each probability.

a. P(all even)

b. P(two clubs and one heart)

Find the mean, median, and mode for each set of data. Then find the range, variance, and standard deviation for each population.

50. {7, 7, 8, 10, 10, 10} **51.** {0.5, 0.4, 0.2, 0.5, 0.2}

LESSON 0-1 Sets

Objective

1. Use set notation to denote elements, subsets, and complements.
2. Find intersections and unions of sets.

NewVocabulary
set
element
subset
universal set
complement
union
intersection
empty set

1 Set Notation

A **set** is a collection of objects. Each object in a set is called an **element**. A set is named using a capital letter and is written with its elements listed within braces { }.

Set Name	Description of Set	Set Notation
C	pages in a chapter of a book	$C = \{35, 36, 37, 38, 39, 40\}$
A	students who made an A on the test	$A =$ {Olinda, Mario, Karen}
L	the letters from A to H	$L =$ {A, B, C, D, E, F, G, H}
N	positive odd numbers	$N = \{1, 3, 5, 7, 9, 11, 13, \ldots\}$

To write that Olinda is *an element* of set A, write Olinda $\in A$.

Example 1 Use Set Notation

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*.

a. N is the set of whole numbers greater than 12 and less than 16. $15 \in N$

The elements in this set are 13, 14, and 15, so $N = \{13, 14, 15\}$. Because 15 is an element of N, $15 \in N$ is a true statement.

b. V is the set of vowels. g $\in V$

The elements in this set are the letters a, e, i, o, and u, so $V =$ {a, e, i, o, u}. Because the letter g is not an element of V, a correct statement is g $\notin V$. Therefore, g $\in V$ is a false statement.

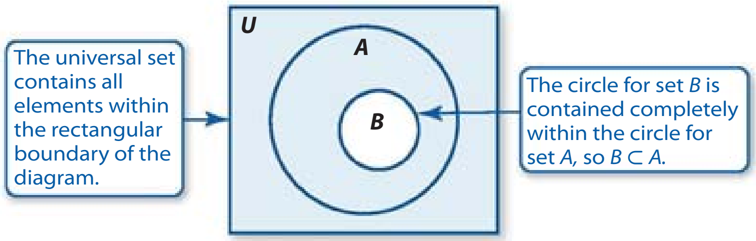
c. M is the set of months that begin with J. April $\in M$

The elements in this set are the months January, June, and July, so $M =$ {January, June, July}. Because the month of April is not an element of this set, a correct statement is April $\notin M$. Therefore, April $\in M$ is a false statement.

d. X is the set of numbers on a die. $4 \in X$

The elements in this set are 1, 2, 3, 4, 5, and 6, so $X = \{1, 2, 3, 4, 5, 6\}$. Because 4 is an element of X, $4 \in X$ is a true statement.

If every element of set B is also contained in set A, then B is called a **subset** of A, and is written as $B \subset A$. The **universal set** U is the set of all possible elements for a situation. All other sets in this situation are subsets of the universal set.



Suppose $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 2, 3\}$. Because $1 \in A$, $2 \in A$, and $3 \in A$, $B \subset A$.

The set of elements in U that are not elements of set B is called the **complement** of B, and is written as B'. In the Venn diagram, the complement of B is all of the shaded regions.

StudyTip

Math Symbols

$\in$	is an element of
$\notin$	is not an element of
$\subset$	is a subset of
A'	the complement of set A
$A \cap B$	the intersection of set A and set B
$A \cup B$	the union of set A and set B

Example 2 Identify Subsets and Complements of Sets

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 4, 5, 7, 8, 9\}$, $B = \{5, 7\}$, $C = \{1, 5, 7, 8\}$, $D = \{2, 3\}$, and $E = \{6, 3\}$.

a. State whether $B \subset A$ is *true* or *false*.

$B = \{5, 7\}$ $\quad$ $A = \{1, 4, 5, 7, 8, 9\}$

True; $5 \in A$ and $7 \in A$, so all of the elements of B are also elements of A. Therefore, B is a subset of A.

b. State whether $E \subset D$ is *true* or *false*.

$E = \{6, 3\}$ $\quad$ $D = \{2, 3\}$

False; $6 \notin D$, so not all of the elements of E are in D. Therefore, E is not a subset of D.

c. Find A'.

Identify the elements of U that are not in A.

$A = \{1, 4, 5, 7, 8, 9\}$ $\quad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $A' = \{0, 2, 3, 6\}$.

d. Find D'.

Identify the elements of U that are not in D.

$D = \{2, 3\}$ $\quad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $D' = \{0, 1, 4, 5, 6, 7, 8, 9\}$.

2 Unions and Intersections

The **union** of sets A and B, written $A \cup B$, is a new set consisting of all of the elements that are in either A or B. The **intersection** of sets A and B, written $A \cap B$, is a new set consisting of elements found in A and B. If two sets have no elements in common, their intersection is called the **empty set**, and is written as $\varnothing$ or $\{\ \}$.

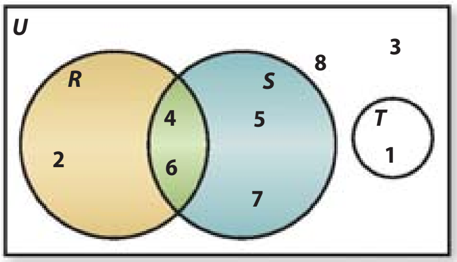
Example 3 Find the Union and Intersection of Two Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $R = \{2, 4, 6\}$, $S = \{4, 5, 6, 7\}$, and $T = \{1\}$.

a. Find $R \cup S$.

The union of R and S is the set of all elements that belong to R, S, or to both sets.

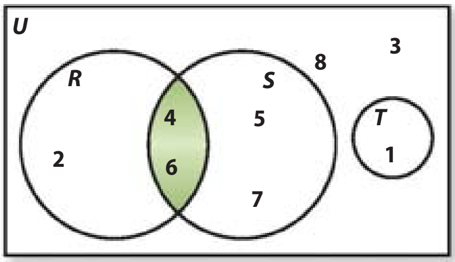
So, $R \cup S = \{2, 4, 5, 6, 7\}$.



b. Find $R \cap S$.

The intersection of R and S is the set of all elements found in both R and S.

So, $R \cap S = \{4, 6\}$.



c. Find $T \cap S$.

Because there are no elements that belong to both T and S, the intersection of T and S is the empty set. So, $T \cap S = \varnothing$.

WatchOut!

Disjoint Sets Sets A and B are said to be *disjoint* if they have no elements in common. The intersection of two disjoint sets is the empty set, while the union of two disjoint sets includes all of the elements from each set.

Exercises

= Step-by-Step Solutions begin on page R29.

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*. (Example 1)

1. J is the set of whole number multiples of 3 that are less than 15. $15 \in J$
2. K is the set of consonant letters in the English alphabet. $h \in K$
3. L is the set of the first six prime numbers. $9 \in L$
4. V is the set of states in the U.S. that border Georgia. Alabama $\notin V$
5. N is the set of natural numbers less than 12. $0 \in N$
6. D is the set of days that start with S. Sunday $\in D$
7. A is the set of girls names that start with A. Ashley $\in A$
8. S is the set of the 48 continental states in the U.S. Hawaii $\notin S$

For Exercises 9–24, use the following information.
Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, $A = \{1, 2, 6, 9, 10, 12\}$, $B = \{2, 9, 10\}$, $C = \{0, 1, 6, 9, 11\}$, $D = \{4, 5, 10\}$, $E = \{2, 3, 6\}$, and $F = \{2, 9\}$.

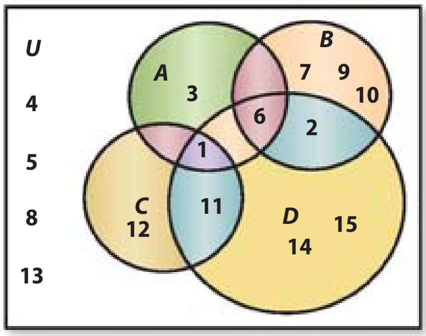
Determine whether each statement is *true* or *false*. Explain your reasoning. (Examples 1 and 2)

9. $3 \in D$
10. $8 \notin A$
11. $B \subset A$
12. $U \subset A$
13. $5 \notin D$
14. $2 \in E$
15. $0 \in F$
16. $6 \notin F$

Find each of the following. (Examples 2 and 3)

17. C'
18. U'
19. A'
20. $D \cap E$
21. $C \cap E$
22. $E \cup F$
23. $A \cup B$
24. $A \cap B$

Use the Venn diagram to find each of the following.
(Examples 2 and 3)



25. $A \cup B$
26. $A \cap D$
27. $C \cup D$
28. A'
29. $A \cap B \cap D$
30. $(A \cup B) \cup C$

31. **SPORTS** Sixteen students in Mr. Frank's gym class each participate in one or more sports as shown in the table. (Examples 2 and 3)

Mr. Frank's Gym Class		
Basketball	**Soccer**	**Volleyball**
Ayanna	Lisa	Pam
Pam	Ayanna	Lisa
Sue	Ron	Shiv
Lisa	Tyron	Max
Ron	Max	Aida
Max	Aida	Juan
Ito	Evita	Tino
Juan	Nelia	Kai
Nelia	Percy	Percy

a. Let B represent the set of basketball players, S represent the set of soccer players, and V represent the set of volleyball players. Draw a Venn diagram of this situation.

b. Find $S \cap V$. What does this set represent?

c. Find S'. What does this set represent?

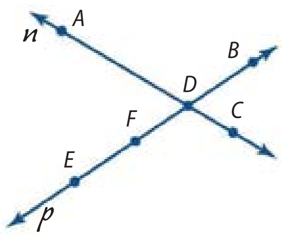
d. Find $B \cup V$. What does this set represent?

32. **ACADEMICS** There are 26 students at West High School who take either calculus or physics or both. Each student is represented by a letter of the alphabet below. Draw a Venn diagram of this situation. (Examples 2 and 3)

Calculus	A, D, F, I, J, K, L, M, P, R, T, V, X, Y, Z
Physics	B, C, D, E, F, G, H, I, J, K, L, N, O, Q, S, U, W

33. **BEVERAGES** Suppose you can select a juice from three possible kinds: apple, orange, or grape, or you can select a soda from two possible kinds, Brand A or Brand B. If you can choose a juice *or* a soda to drink, according to the Addition Principle of Counting, you have $3 + 2$ or 5 possible choices. Using notation that you have learned in this lesson, justify this result. In what situation could this principle not be applied?

GEOMETRY Use the figure to find the simplest name for each of the following.



34. $\overline{DE} \cap \overline{BF}$
35. $\overline{AD} \cup \overline{DC}$
36. $\overrightarrow{DE} \cup \overrightarrow{DC}$
37. line n $\cap$ line p
38. $\overleftrightarrow{AC} \cap \overline{EF}$
39. $\overrightarrow{FB} \cup \overrightarrow{EB}$

LESSON 0-2 Operations with Complex Numbers

Objective

1 Perform operations with pure imaginary numbers and complex numbers.

2 Use complex conjugates to write quotients of complex numbers in standard form.

NewVocabulary

imaginary unit
complex number
standard form
real part
imaginary part
imaginary number
pure imaginary number
complex conjugates

1 Imaginary and Complex Numbers

The **imaginary unit *i*** is defined as the principal square root of -1 and can be written as $i = \sqrt{-1}$. The first eight powers of i are shown below.

$$i^1 = i \qquad i^5 = i^4 \cdot i = i$$
$$i^2 = -1 \qquad i^6 = i^4 \cdot i^2 = -1$$
$$i^3 = i^2 \cdot i = -i \qquad i^7 = i^4 \cdot i^3 = -i$$
$$i^4 = i^2 \cdot i^2 = 1 \qquad i^8 = i^4 \cdot i^4 = 1$$

Notice that the pattern $i, -1, -i, 1, \ldots$ repeats after the first four results. In general, the value of i^n, where n is a whole number, can be found by dividing n by 4 and examining the remainder.

KeyConcept Powers of *i*

To find the value of i^n, let R be the remainder when n is divided by 4.

$n < 0$	$n > 0$
$R = -3 \rightarrow i^n = i$	$R = 1 \rightarrow i^n = i$
$R = -2 \rightarrow i^n = -1$	$R = 2 \rightarrow i^n = 1$
$R = -1 \rightarrow i^n = -i$	$R = 3 \rightarrow i^n = -i$
$R = 0 \rightarrow i^n = 1$	$R = 0 \rightarrow i^n = 1$

Example 1 Powers of *i*

Simplify.

a. i^{53}

Method 1

$53 \div 4 = 13 \text{ R } 1$

If $R = 1$, $i^n = i$.

$i^{53} = i$

Method 2

$$\begin{aligned} i^{53} &= (i^4)^{13} \cdot i \\ &= (1)^{13} \cdot i \\ &= i \end{aligned}$$

b. i^{-18}

Method 1

$-18 \div 4 = -4 \text{ R } -2$

If $R = -2$, $i^n = -1$.

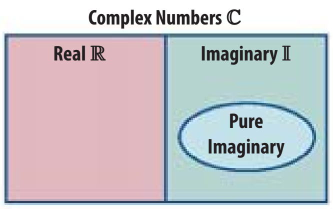
$i^{-18} = -1$

Method 2

$$\begin{aligned} i^{-18} &= (i^4)^{-5} \cdot i^2 \\ &= (1)^{-5} \cdot (-1) \\ &= -1 \end{aligned}$$

Sums of real numbers and real number multiples of i belong to an extended system of numbers, known as *complex numbers*. A **complex number** is a number that can be written in the **standard form** $a + bi$, where the real number a is the **real part** and the real number b is the **imaginary part**.

If $a \neq 0$ and $b = 0$, the complex number is $a + 0i$, or the real number a. Therefore, all real numbers are also complex numbers. If $b \neq 0$ the complex number is known as an **imaginary number**. If $a = 0$ and $b \neq 0$, such as $4i$ or $-9i$, the complex number is a **pure imaginary number**.



Complex numbers can be added and subtracted by performing the chosen operation on the real and imaginary parts separately.

Example 2 Add and Subtract Complex Numbers

Simplify.

a. $(5 - 3i) + (-2 + 4i)$

$$(5 - 3i) + (-2 + 4i) = [5 + (-2)] + (-3i + 4i)$$ Group the real and imaginary parts together.

$$= 3 + i$$ Simplify.

b. $(10 - 2i) - (14 - 6i)$

$$(10 - 2i) - (14 - 6i) = 10 - 2i - 14 + 6i$$ Distributive Property

$$= -4 + 4i$$ Simplify.

Many properties of real numbers, such as the Distributive Property, are also valid for complex numbers. Because of this, complex numbers can be multiplied using similar techniques to those that are used when multiplying binomials.

Technology Tip

Complex Numbers Some calculators have a complex number mode. In this mode, they can perform complex number arithmetic.

Example 3 Multiply Complex Numbers

Simplify.

a. $(2 - 3i)(7 - 4i)$

$$(2 - 3i)(7 - 4i) = 14 - 8i - 21i + 12i^2$$ FOIL method

$$= 14 - 8i - 21i + 12(-1)$$ $i^2 = -1$

$$= 2 - 29i$$ Simplify.

b. $(4 + 5i)(4 - 5i)$

$$(4 + 5i)(4 - 5i) = 16 - 20i + 20i - 25i^2$$ FOIL method

$$= 16 - 25(-1)$$ $i^2 = -1$

$$= 41$$ Simplify.

2 Use Complex Conjugates

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. Complex conjugates can be used to rationalize an imaginary denominator. Multiply the numerator and the denominator by the complex conjugate of the denominator.

Watch Out!

Complex Conjugates Make sure that you change the sign of the imaginary part of the complex number to find the complex conjugate.

Example 4 Rationalize a Complex Expression

Simplify $(5 - 3i) \div (1 - 2i)$.

$$(5 - 3i) \div (1 - 2i) = \frac{5 - 3i}{1 - 2i}$$ Rewrite as a fraction.

$$= \frac{5 - 3i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i}$$ Multiply the numerator and denominator by the conjugate of the denominator, $1 + 2i$.

$$= \frac{5 + 10i - 3i - 6i^2}{1 - 4i^2}$$ Multiply.

$$= \frac{5 + 7i - 6(-1)}{1 - 4(-1)}$$ $i^2 = -1$

$$= \frac{11 + 7i}{5}$$ Simplify.

$$= \frac{11}{5} + \frac{7}{5}i$$ Write the answer in the form $a + bi$.

Exercises

● = Step-by-Step Solutions begin on page R29.

Simplify. (Example 1)

1. i^{-10}

2. $i^2 + i^8$

3. $i^3 + i^{20}$

4. i^{100}

5. i^{77}

6. $i^4 + i^{-12}$

7. $i^5 + i^9$

8. i^{18}

Simplify. (Example 2)

9. $(3 + 2i) + (-4 + 6i)$

10. $(7 - 4i) + (2 - 3i)$

11. $(0.5 + i) - (2 - i)$

12. $(-3 - i) - (4 - 5i)$

13. $(2 + 4.1i) - (-1 - 6.3i)$

14. $(2 + 3i) + (-6 + i)$

15. $(-2 + 4i) + (5 - 4i)$

16. $(5 + 7i) - (-5 + i)$

17. ELECTRICITY Engineers use imaginary numbers to express the two-dimensional quantity of alternating current, which involves both amplitude and angle. In these imaginary numbers, i is replaced with j because engineers use I as a variable for the entire quantity of current. *Impedance* is the measure of how much hinderance there is to the flow of the charge in a circuit with alternating current. The impedance in one part of a series circuit is $2 + 5j$ ohms and the impedance in another part of the circuit is $7 - 3j$ ohms. Add these complex numbers to find the total impedance in the circuit. (Example 2)

Simplify. (Example 3)

18. $(-2 - i)^2$

19. $(1 + 4i)^2$

20. $(5 + 2i)^2$

21. $(3 + i)^2$

22. $(2 + i)(4 + 3i)$

23. $(3 + 5i)(3 - 5i)$

24. $(5 + 3i)(2 + 6i)$

25. $(6 + 7i)(6 - 7i)$

Simplify. (Example 4)

26. $\dfrac{5 + i}{6 + i}$

27. $\dfrac{i}{1 + 2i}$

28. $\dfrac{5 - i}{5 + i}$

29. $\dfrac{3 - 2i}{-4 - i}$

30. $\dfrac{1 + 2i}{2 - 3i}$

31. $\dfrac{3 + 4i}{1 + 5i}$

32. $\dfrac{2 - \sqrt{2}i}{3 + \sqrt{6}i}$

33. $\dfrac{1 + \sqrt{3}i}{1 - \sqrt{2}i}$

ELECTRICITY The voltage E, current I, and impedance Z in a circuit are related by $E = I \cdot Z$. Find the voltage (in volts) in each of the following circuits given the current and impedance.

34. $I = 1 + 3j$ amps, $Z = 7 - 5j$ ohms

35. $I = 2 - 7j$ amps, $Z = 4 - 3j$ ohms

36. $I = 5 - 4j$ amps, $Z = 3 + 2j$ ohms

37. $I = 3 + 10j$ amps, $Z = 6 - j$ ohms

Solve each equation.

38. $5x^2 + 5 = 0$

39. $4x^2 + 64 = 0$

40. $2x^2 + 12 = 0$

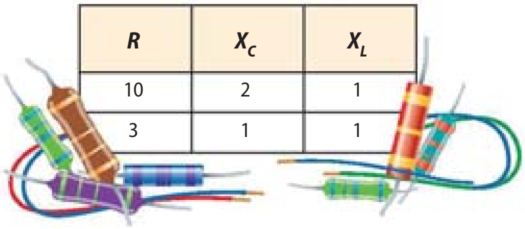
41. $6x^2 + 72 = 0$

42. $8x^2 + 120 = 0$

43. $3x^2 + 507 = 0$

44. ELECTRICITY The impedance Z of a circuit depends on the resistance R, the reactance due to capacitance X_C, and the reactance due to inductance X_L, and can be written as a complex number $R + (X_L - X_C)j$. The values (in ohms) for R, X_C, and X_L in the first and second parts of a particular series circuit are shown.

Series Circuit



R	X_C	X_L
10	2	1
3	1	1

a. Write complex numbers that represent the impedances in the two parts of the circuit.

b. Add your answers from part **a** to find the total impedance in the circuit.

c. The *admittance* S of a circuit is the measure of how easily the circuit allows current to flow and is the reciprocal of impedance. Find the admittance (in siemens) in a circuit with an impedance of $6 + 3j$ ohms.

Find values of x and y to make each equation true.

45. $3x + 2iy = 6 + 10i$

46. $5x + 3iy = 5 - 6i$

47. $x - iy = 3 + 4i$

48. $-5x + 3iy = 10 - 9i$

49. $2x + 3iy = 12 + 12i$

50. $4x - iy = 8 + 7i$

Simplify.

51. $(2 - i)(3 + 2i)(1 - 4i)$

52. $(-1 - 3i)(2 + 2i)(1 - 2i)$

53. $(2 + i)(1 + 2i)(3 - 4i)$

54. $(-5 - i)(6i + 1)(7 - i)$

LESSON 0-3 Quadratic Functions and Equations

Objective

1. Graph quadratic functions.
2. Solve quadratic equations.

NewVocabulary
parabola
axis of symmetry
vertex
quadratic equation
completing the square
Quadratic Formula

1 Graph Quadratic Functions

The graph of a quadratic function is called a **parabola**. To graph a quadratic function, graph ordered pairs that satisfy the function.

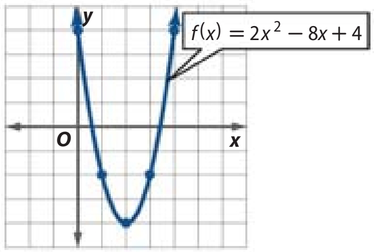
Example 1 Graph a Quadratic Function Using a Table

Graph $f(x) = 2x^2 - 8x + 4$ by making a table of values.

Step 1 Choose integer values for x and evaluate the function for each value.

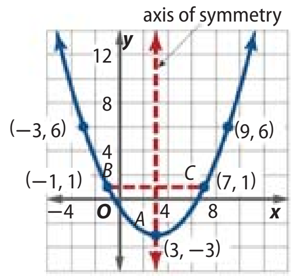
x	$f(x) = 2x^2 - 8x + 4$	$f(x)$	$(x, f(x))$
0	$f(0) = 2(0)^2 - 8(0) + 4$	4	(0, 4)
1	$f(1) = 2(1)^2 - 8(1) + 4$	−2	(1, −2)
2	$f(2) = 2(2)^2 - 8(2) + 4$	−4	(2, −4)
3	$f(3) = 2(3)^2 - 8(3) + 4$	−2	(3, −2)
4	$f(4) = 2(4)^2 - 8(4) + 4$	4	(4, 4)

Step 2 Graph the resulting coordinate pairs, and connect the points with a smooth curve.



The **axis of symmetry** is a line that divides the parabola into two halves that are reflections of each other. Notice that, because the parabola is symmetric about the axis of symmetry, points B and C are 4 units from the x-coordinate of the vertex, and they have the same y-coordinate.

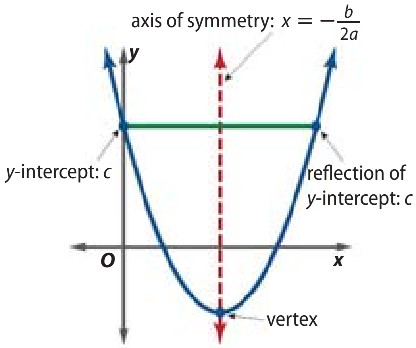
The axis of symmetry intersects a parabola at a point called the **vertex**. The vertex of the graph at the right is $A(3, -3)$.



KeyConcept Graph of a Quadratic Function

Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The y-intercept is $a(0)^2 + b(0) + c$ or c.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.



Example 2 Axis of Symmetry, *y*-intercept, and Vertex

Use the axis of symmetry, *y*-intercept, and vertex to graph $f(x) = x^2 + 2x - 3$.

Step 1 Determine a, b, and c.

$$f(x) = ax^2 + bx + c$$

$$f(x) = 1x^2 + 2x - 3$$

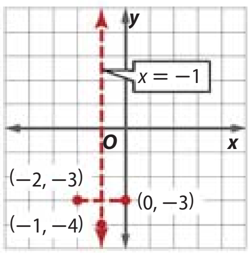
$a = 1$, $b = 2$, and $c = -3$

Step 2 Use a and b to find the equation of the axis of symmetry.

$x = -\frac{b}{2a}$ — Equation of the axis of symmetry

$= -\frac{2}{2(1)}$ or -1 — $a = 1$ and $b = 2$

Step 3 Find the coordinates of the vertex.



Because the equation of the axis of symmetry is $x = -1$, the x-coordinate of the vertex is -1.

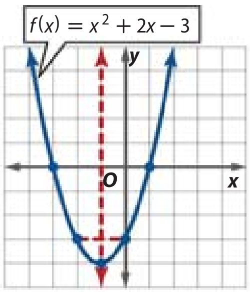
$f(x) = x^2 + 2x - 3$ — Original equation

$f(-1) = (-1)^2 + 2(-1) - 3$ — Evaluate $f(x)$ for $x = -1$.

$f(-1) = -4$ — Simplify.

The vertex is at $(-1, -4)$.

Step 4 Find the y-intercept and its reflection.



Because $c = -3$, the coordinates of the y-intercept are $(0, -3)$. The axis of symmetry is $x = -1$, so the reflection of the y-intercept is $(-2, -3)$.

Step 5 Graph the axis of symmetry, vertex, y-intercept and its reflection. Find and graph one or more additional points and their reflections. Then connect the points with a smooth curve.

StudyTip

Graphing Quadratic Functions You can always use a table of values to generate more points for the graph of a quadratic function.

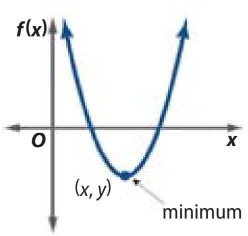
The y-coordinate of the vertex of a quadratic function is the *maximum* or *minimum value* of the function. These values represent the greatest or least possible value that the function can reach.

KeyConcept Maximum and Minimum Values

The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$

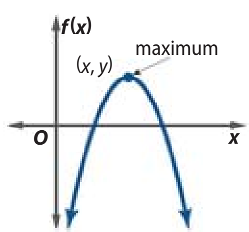
- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

$a > 0$



The y-coordinate of the vertex is the minimum value.

$a < 0$

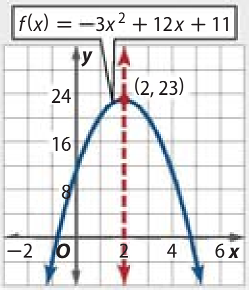


The y-coordinate of the vertex is the maximum value.

The domain of a quadratic function is all real numbers. The range will either be all real numbers less than or equal to the maximum value or all real numbers greater than or equal to the minimum value.

StudyTip

Check Your Answers Graphically You can check your answers in Example 3 by graphing $f(x) = -3x^2 + 12x + 11$.



From the graph, you can see that the maximum value of the function is $y = 23$, the domain is $\mathbb{R}$, and the range is $y \leq 23$, for $y \in \mathbb{R}$.

Example 3 Maximum and Minimum Values

Consider $f(x) = -3x^2 + 12x + 11$.

a. Determine whether the function has a *maximum* or *minimum* value.

For this function, $a = -3$. Because a is negative, the graph opens down and the function has a maximum value.

b. Find the maximum or minimum value of the function.

The maximum value of the function is the y-coordinate of the vertex. To find the coordinates of the vertex, first find the equation of the axis of symmetry.

$x = -\frac{b}{2a}$	Equation of the axis of symmetry
$= -\frac{12}{2(-3)}$	$a = -3$ and $b = 12$
$= 2$	Simplify.

Because the equation of the axis of symmetry is $x = 2$, the x-coordinate of the vertex is 2. You can now find the y-coordinate of the vertex, or maximum value of the function, by evaluating the original function for $x = 2$.

$f(x) = -3x^2 + 12x + 11$	Original function
$f(2) = -3(2)^2 + 12(2) + 11$	$x = 2$
$= -12 + 24 + 11$	Multiply.
$= 23$	Simplify.

Therefore, $f(x)$ has a maximum value of 23 at $x = 2$.

c. State the domain and range of the function.

The domain of the function is all real numbers or $\mathbb{R}$. The range of the function is all real numbers less than or equal to the maximum value 23 or $y \leq 23$.

2 Solve Quadratic Equations

A **quadratic equation** is a polynomial equation of degree 2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$. The factors of a quadratic polynomial can be used to solve the related quadratic equation. Solving quadratic equations by factoring is an application of the Zero Product Property.

KeyConcept Zero Product Property

For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

Example 4 Solve by Factoring

Solve $x^2 - 8x + 12 = 0$ by factoring.

$x^2 - 8x + 12 = 0$	Original equation
$(x - 2)(x - 6) = 0$	Factor.
$x - 2 = 0 \quad \text{or} \quad x - 6 = 0$	Zero Product Property
$x = 2 \qquad\qquad x = 6$	Simplify.

The solutions of the equation are 2 and 6.

Another method for solving quadratic equations is to **complete the square**.

KeyConcept Complete the Square

To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example 5 Solve by Completing the Square

Solve $x^2 - 4x + 1 = 0$ by completing the square.

$x^2 - 4x + 1 = 0$	Original equation
$x^2 - 4x = -1$	Rewrite so that the left side is of the form $x^2 + bx$.
$x^2 - 4x + 4 = -1 + 4$	Because $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = 3$	Write the left side as a perfect square.
$x - 2 = \pm\sqrt{3}$	Take the square root of each side.
$x = 2 \pm \sqrt{3}$	Add 2 to each side.
$x = 2 + \sqrt{3}$ or $x = 2 - \sqrt{3}$	Write as two equations.
≈ 3.73 $\qquad$ ≈ 0.27	Use a calculator.

The solutions of the equation are approximately 0.27 and 3.73.

WatchOut!

Completing the Square When completing the square, the coefficient of the x^2 term must be 1.

Completing the square can be used to develop a general formula that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$, known as the **Quadratic Formula**.

KeyConcept Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 6 Solve by Using the Quadratic Formula

Solve $x^2 - 4x + 15 = 0$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(15)}}{2(1)}$	Replace a with 1, b with -4, and c with 15.
$= \frac{4 \pm \sqrt{16 - 60}}{2}$	Multiply.
$= \frac{4 \pm \sqrt{-44}}{2}$	Simplify.
$= \frac{4 \pm 2i\sqrt{11}}{2}$	$\sqrt{-44} = \sqrt{4}\,\sqrt{-1}\,\sqrt{11}$ or $2i\sqrt{11}$
$= 2 \pm \sqrt{11}i$	Simplify.

The solutions are $2 + \sqrt{11}i$ and $2 - \sqrt{11}i$.

StudyTip

Discriminant The expression $b^2 - 4ac$ is called the *discriminant* and is used to determine the number and types of roots of a quadratic equation. For example, when the discriminant is 0, there are two rational roots.

Graph each equation by making a table of values. (Example 1)

1. $f(x) = x^2 + 5x + 6$

2. $f(x) = x^2 - x - 2$

3. $f(x) = 2x^2 + x - 3$

4. $f(x) = 3x^2 + 4x - 5$

5. $f(x) = x^2 - x - 6$

6. $f(x) = -x^2 - 3x - 1$

7. BASEBALL A batter hits a baseball with an initial speed of 80 feet per second. If the initial height of the ball is 3.5 feet above the ground, the function $d(t) = 80t - 16t^2 + 3.5$ models the ball's height above the ground in feet as a function of time in seconds. Graph the function using a table of values. (Example 1)

Use the axis of symmetry, y-intercept, and vertex to graph each function. (Example 2)

8. $f(x) = x^2 + 3x + 2$

9. $f(x) = x^2 - 9x + 8$

10. $f(x) = x^2 - 2x + 1$

11. $f(x) = x^2 - 6x - 16$

12. $f(x) = 2x^2 - 8x - 5$

13. $f(x) = 3x^2 + 12x - 4$

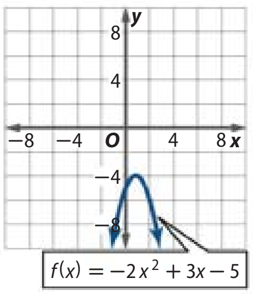
14. HEALTH The normal systolic pressure P in millimeters of mercury (mm Hg) for a woman can be modeled by $P(x) = 0.01x^2 + 0.05x + 107$, where x is age in years. (Example 2)

a. Find the axis of symmetry, y-intercept, and vertex for the graph of $P(x)$.

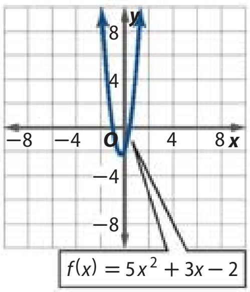
b. Graph $P(x)$ using the values you found in part **a**.

Determine whether each function has a *maximum* or *minimum* value. Then find the value of the maximum or minimum, and state the domain and range of the function. (Example 3)

15.



16.



17. $f(x) = -x^2 + 3x - 5$

18. $f(x) = x^2 - 5x + 6$

19. $f(x) = 2x^2 + 4x + 7$

20. $f(x) = 6x^2 + 3x - 1$

21. $f(x) = -3x^2 - 2x - 1$

22. $f(x) = -5x^2 + 10x - 6$

Solve each equation by factoring. (Example 4)

23. $x^2 - 10x + 21 = 0$

24. $p^2 - 6p + 5 = 0$

25. $x^2 - 3x - 28 = 0$

26. $4w^2 + 19w - 5 = 0$

27. $4r^2 - r = 5$

28. $g^2 + 6g - 16 = 0$

Solve each equation by completing the square. (Example 5)

29. $x^2 + 8x - 20 = 0$

30. $2a^2 + 11a - 21 = 0$

31. $z^2 - 2z - 24 = 0$

32. $p^2 - 3p - 88 = 0$

33 $t^2 - 3t - 7 = 0$

34. $3g^2 - 12g = -4$

Solve each equation by using the Quadratic Formula. (Example 6)

35. $m^2 + 12m + 36 = 0$

36. $t^2 - 6t + 13 = 0$

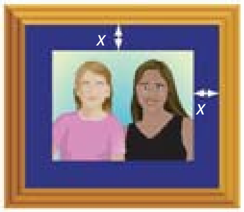
37. $6m^2 + 7m - 3 = 0$

38. $c^2 - 5c + 9 = 0$

39. $4x^2 - 2x + 9 = 0$

40. $3p^2 + 4p = 8$

41. PHOTOGRAPHY Jocelyn wants to frame a cropped photograph that has an area of 20 square inches with a uniform width of matting between the photograph and the edge of the frame as shown.



a. Write an equation to model the situation if the length and width of the matting must be 8 inches by 10 inches, respectively, to fit in the frame.

b. Graph the related function.

c. What is the width of the exposed part of the matting x?

Solve each equation.

42. $x^2 + 5x - 6 = 0$

43. $a^2 - 13a + 40 = 0$

44. $x^2 - 11x + 24 = 0$

45. $q^2 - 12q + 36 = 0$

46. $-x^2 + 4x - 6 = 0$

47. $7x^2 + 3 = 0$

48. $x^2 - 4x + 7 = 0$

49. $2x^2 + 6x - 3 = 0$

50. PETS A rectangular turtle pen is 6 feet long by 4 feet wide. The pen is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new pen?

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

51. Their sum is -17 and their product is 72.

52. Their sum is 7 and their product is 14.

53. Their sum is -9 and their product is 24.

54. Their sum is 12 and their product is -28.

LESSON 0-4

nth Roots and Real Exponents

Objective

1. Simplify expressions in radical form.
2. Simplify expressions in exponential form.

NewVocabulary
nth root
principal nth root

1 Simplify Radicals A square root of a number is one of two equal factors of that number. For example, 4 is a square root of 16 because $4 \cdot 4$ or $4^2 = 16$. In general, if a and b are real numbers and n is a positive integer greater than 1, if $b^n = a$, then b is an **nth root** of a.

KeyConcept nth Root of a Number

Let a and b be real numbers and let n be any positive integer greater than 1.

- If $a = b^n$, then b is an nth root of a.
- If a has an nth root, the **principal nth root** of a is the root having the same sign as a.

The principal nth root of a is denoted by the radical expression $\sqrt[n]{a}$, where n is the index of the radical and a is the radicand.

Some examples of nth roots are listed below.

$\sqrt[4]{81} = 3$ — $\sqrt[4]{81}$ indicates the principal fourth root of 81.

$-\sqrt[4]{81} = -3$ — $-\sqrt[4]{81}$ indicates the opposite of the principal fourth root of 81.

$\pm\sqrt[4]{81} = \pm 3$ — $\pm\sqrt[4]{81}$ indicates both real fourth roots of 81.

Whether a radical expression has positive and/or negative roots is dependent upon the value of the radicand and whether the index is even or odd.

Real nth Roots of Real Numbers

Suppose n is an integer greater than 1, and a is a real number.

a	n is even.	n is odd.
$a > 0$	1 positive and 1 negative real root: $\pm\sqrt[n]{a}$	1 positive and 0 negative real root: $\sqrt[n]{a}$
$a < 0$	0 real roots	0 positive and 1 negative real root: $\sqrt[n]{a}$
$a = 0$	1 real root: $\sqrt[n]{0} = 0$	1 real root: $\sqrt[n]{0} = 0$

Example 1 Find nth Roots

Evaluate.

a. $-\sqrt{49}$

$-\sqrt{49} = -(\sqrt{49})$ or -7 — Simplify.

b. $\sqrt[3]{\frac{64}{27}}$

$\sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}}$ or $\frac{4}{3}$ — Simplify.

c. $\sqrt[4]{-121}$

Because there is no real number that can be raised to the fouth power to produce -121, $\sqrt[4]{-121}$ is not a real number.

KeyConcept Basic Properties of Radicals

Let a and b be real numbers, variables, or algebraic expressions, and n be a positive integer greater than 1, where all of the roots are real numbers and all of the denominators are greater than 0. Then the following properties are true.

Product Property $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Quotient Property $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

When you find an even root of an even power and the result is an odd power, you must use the absolute value of the result to ensure that the answer is nonnegative.

Example 2 Simplify Using Absolute Value

Simplify.

a. $\sqrt[6]{n^{18}}$

$$\sqrt[6]{n^{18}} = \sqrt[6]{(n^3)^6}$$
$$= |n^3|$$

Notice that n^3 is a sixth root of n^{18}. Because the index is even and the exponent is odd, you must use the absolute value of n^3.

b. $\sqrt[4]{81(a+1)^{12}}$

$$\sqrt[4]{81(a+1)^{12}} = \sqrt[4]{[3(a+1)^3]^4}$$
$$= 3|(a+1)^3|$$

Because the index is even and the exponent is odd, you must use the absolute value of $(a+1)^3$.

c. $\sqrt{63y^3}$

$$\sqrt{63y^3} = \sqrt{9y^2 \cdot 7y}$$
$$= \sqrt{(3y)^2} \cdot \sqrt{7y}$$
$$= 3y\sqrt{7y}$$

Notice that $\sqrt{63y^3}$ is only a real number when y is nonnegative. Therefore, it is not necessary to use absolute value.

d. $\sqrt[5]{-p^{10}q^7}$

$$\sqrt[5]{-p^{10}q^7} = \sqrt[5]{-1p^{10}q^5 \cdot q^2}$$
$$= \sqrt[5]{(-1p^2q)^5} \cdot \sqrt[5]{q^2}$$
$$= -p^2q\sqrt[5]{q^2}$$

Because the index is odd, it is not necessary to use absolute value.

StudyTip

Odd Index If n is odd, there is only one real root. Therefore, absolute value symbols are never needed.

2 Rational Exponents

Squaring a number and taking the square root of a number are inverse operations. This relationship can be used to express radicals in exponential form.

Let $\sqrt{b} = b^n$.

$\sqrt{b} = b^n$	Given
$(\sqrt{b})^2 = (b^n)^2$	Square each side.
$b = b^{2n}$	Simplify.
$1 = 2n$	If $a^m = a^n$ then $m = n$.
$\frac{1}{2} = n$	Divide each side by 2.

So, $\sqrt{b} = b^{\frac{1}{2}}$. This process can be used to determine the exponential form for any nth root. You can determine the exponential form for any nth root using the properties shown below.

KeyConcept Rational Exponents

If b is a real number, variable, or algebraic expression and m and n are positive integers greater than 1, then

- $b^{\frac{1}{n}} = \sqrt[n]{b}$, if the principal nth root of b exists, and
- $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$ or $\sqrt[n]{b^m}$, if $\frac{m}{n}$ is in reduced form.

WatchOut!

Common Error Be careful not to confuse rational and negative exponents.

$b^{-n} \neq b^{\frac{1}{n}}$

$b^{-n} = \frac{1}{b^n}$

$b^{\frac{1}{n}} = \sqrt[n]{b}$

Example 3 Simplify Expressions with Rational Exponents

Simplify.

a. $\dfrac{\sqrt[10]{32}}{\sqrt[6]{2}}$

$\dfrac{\sqrt[10]{32}}{\sqrt[6]{2}} = \dfrac{32^{\frac{1}{10}}}{2^{\frac{1}{6}}}$	$\sqrt[n]{b} = b^{\frac{1}{n}}$
$= \dfrac{(2^5)^{\frac{1}{10}}}{2^{\frac{1}{6}}}$	$32 = 2^5$
$= \dfrac{2^{\frac{1}{2}}}{2^{\frac{1}{6}}}$	Power of a Power
$= 2^{\frac{1}{2} - \frac{1}{6}}$	Quotient of Powers
$= 2^{\frac{1}{3}}$ or $\sqrt[3]{2}$	Simplify.

b. $\sqrt[4]{x^{21}y^{15}}$

$\sqrt[4]{x^{21}y^{15}} = \sqrt[4]{x^{21}} \cdot \sqrt[4]{y^{15}}$	Product of Roots
$= x^{\frac{21}{4}} \cdot y^{\frac{15}{4}}$	$\sqrt[n]{b^m} = b^{\frac{m}{n}}$
$= x^{\frac{20}{4}} \cdot x^{\frac{1}{4}} \cdot y^{\frac{12}{4}} \cdot y^{\frac{3}{4}}$	Product of Powers
$= x^5 y^3 \sqrt[4]{xy^3}$	Simplify.

c. $\dfrac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}}$

$\dfrac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}} = 16^{\frac{3}{4} - \frac{1}{4}}$	Quotient of Powers
$= 16^{\frac{1}{2}}$	Simplify.
$= \sqrt{16}$ or 4	$b^{\frac{1}{n}} = \sqrt[n]{b}$

Exercises

= Step-by-Step Solutions begin on page R29.

Evaluate. (Example 1)

1. $-\sqrt{169}$
2. $\sqrt{-100}$
3. $\sqrt[3]{\frac{216}{125}}$
4. $\sqrt[3]{-\frac{64}{343}}$
5. $\sqrt[4]{-81}$
6. $\sqrt[4]{625}$
7. $\sqrt[5]{243}$
8. $\sqrt[5]{-1024}$

Simplify. (Example 2)

9. $\sqrt[3]{-27x^9}$
10. $\sqrt[4]{16a^{20}}$
11. $\sqrt[8]{8y^{16}}$
12. $\sqrt[3]{54x^{17}}$
13. $\sqrt{20x^{16}}$
14. $\sqrt{121(z-2)^{14}}$
15. $\sqrt[4]{a^{12}b^9}$
16. $\sqrt[7]{-q^{13}r^{16}}$

Simplify. (Example 3)

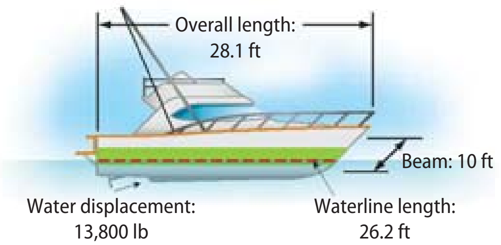
17. $\dfrac{b^{\frac{5}{4}} \cdot b^{\frac{3}{4}}}{b^{\frac{1}{4}}}$
18. $\left(2x^{\frac{1}{4}}y^{\frac{1}{3}}\right)\left(3x^{\frac{1}{4}}y^{\frac{2}{3}}\right)$
19. $\sqrt[6]{640a^3}$
20. $\sqrt[6]{128b^4}$
21. $\dfrac{\sqrt[3]{16}}{\sqrt[5]{4}}$
22. $\dfrac{\sqrt[4]{27}}{\sqrt[3]{81}}$

23. **BOATING** The motion comfort ratio M of a boat is given by

$$M = \frac{D}{0.65(B)^{\frac{4}{3}}(0.7W + 0.3A)},$$

where D is the water displacement of the boat in pounds, B is the boat's beam or width in feet, W is the boat's length in feet at the waterline, and A is the boat's overall length in feet. The higher the ratio, the greater the level of comfort experienced by those on board as the boat encounters waves. (Example 3)

a. Find the motion comfort ratio of the boat shown below.



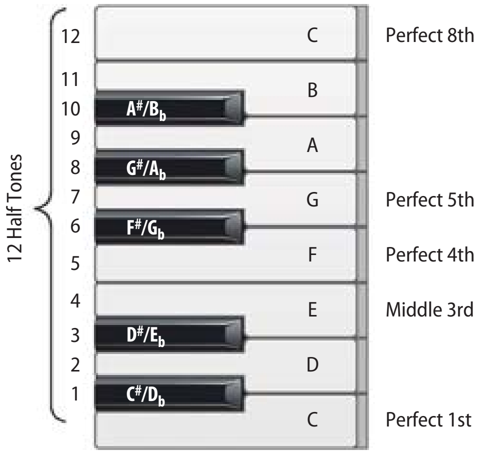
b. Find the beam of a boat to the nearest foot with a comfort ratio of 27 that displaces 15,000 pounds of water, has a waterline length of 30.4 feet, and an overall length of 32.3 feet.

24. **CARS** The value of a car depreciates or declines over the course of its useful life. The new value V and the original value v of a car are related by the formula $V = v(1-r)^n$, where r is the rate of depreciation per year and n is the number of years. Suppose the current value of a used car is $12,000. What would be the value of the car after 18 months at an annual depreciation rate of 20%? (Example 3)

Evaluate.

25. $216^{\frac{1}{3}}$
26. $4096^{\frac{1}{4}}$
27. $49^{-\frac{1}{2}}$
28. $27^{-\frac{1}{3}}$

29. **MUSIC** The note progression of the twelve tone scale is comprised of a series of half tones. In order for an instrument to be "in tune," the frequency of each note has an optimum ratio with the frequency of middle C, called the perfect 1st.



The optimum frequency ratio r, expressed as a decimal, can be calculated using $r = \left(\sqrt[12]{2}\right)^n$, where n is the number of half tones the note is above the perfect 1st, including the note itself. (Example 1)

a. Approximate the optimum frequency ratio of the middle 3rd with the perfect 1st.

b. Without the use of a calculator, approximate the optimum frequency ratio of the perfect 8th and the perfect 1st. Justify your answer.

Simplify.

30. $\sqrt[3]{-250r^{11}t^6u^5}$
31. $\sqrt[3]{128a^9b^7c^4}$
32. $\sqrt[4]{96a^8b^6c^{20}}$
33. $\sqrt[6]{64x^7y^6z^{18}}$
34. $\sqrt[4]{a^2b^3c^4d^5}$
35. $\sqrt[5]{w^6x^8y^{10}z^{13}}$

LESSON 0-5 Systems of Linear Equations and Inequalities

Objective

1. Use various techniques to solve systems of equations.
2. Solve systems of inequalities by graphing.

NewVocabulary

system of equations
substitution method
elimination method
consistent
independent
dependent
inconsistent
system of inequalities

1 Systems of Equations A **system of equations** is a set of two or more equations. To *solve* a system of equations means to find values for the variables in the equations that make all of the equations true. One way to solve a system of equations is by graphing the equations on the same coordinate plane. The point of intersection of the graphs of the equations represents the solution of the system.

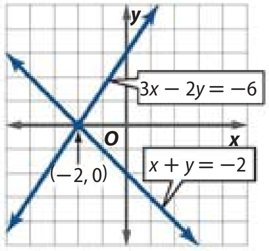
Example 1 Solve by Graphing

Solve the system of equations by graphing.

$3x - 2y = -6$
$x + y = -2$

Solve each equation for y. Then graph each equation.

$y = \frac{3}{2}x + 3 \qquad y = -x - 2$



The lines intersect at the point $(-2, 0)$. This ordered pair is the solution of the system.

CHECK	$3x - 2y = -6$	Original equations	$x + y = -2$
	$3(-2) - 2(0) \stackrel{?}{=} -6$	$x = -2$ and $y = 0$	$-2 + 0 \stackrel{?}{=} -2$
	$-6 = -6$ ✓	Simplify.	$-2 = -2$ ✓

Algebraic methods are used to find exact solutions of systems of equations. One algebraic method is called the **substitution method**.

KeyConcept Substitution Method

Step 1 Solve one equation for one of the variables in terms of the other.

Step 2 Substitute the expression found in Step 1 into the other equation to obtain an equation in one variable. Then solve the equation.

Step 3 Substitute to solve for the other variable.

Example 2 Solve by Substitution

Use substitution to solve the system of equations.

$2x + 3y = 9$
$5x - y = 14$

Step 1 $5x - y = 14 \rightarrow y = 5x - 14$ Solve for y.

Step 2

$2x + 3y = 9$	First equation
$2x + 3(5x - 14) = 9$	Substitute $5x - 14$ for y.
$17x - 42 = 9$	Simplify.
$x = 3$	Solve for x.

Step 3

$y = 5x - 14$	Step 1 equation
$= 5(3) - 14$ or 1	The solution is $(3, 1)$.

CHECK From the graph in Figure 0.5.1, you can see that the lines intersect at the point $(3, 1)$.

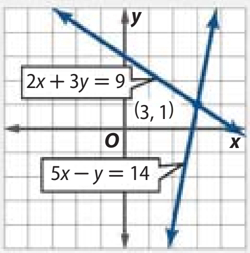


Figure 0.5.1

You can use the **elimination method** to solve a system when one of the variables has the same coefficient in both equations.

KeyConcept Elimination Method

Step 1 Multiply one or both equations by a number to result in two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute to solve for the other variable.

Example 3 Solve by Elimination

Use elimination to solve the system of equations.

$\mathbf{1.5x + 2y = 20}$

$\mathbf{2.5x - 5y = -25}$

$1.5x + 2y = 20$ — Multiply by 5. → $7.5x + 10y = 100$

$2.5x - 5y = -25$ — Multiply by 2. → $(+)\ [5x - 10y = -50]$

$12.5x = 50$ Add.

$x = 4$ Divide each side by 12.5

$1.5x + 2y = 20$ Equation 1

$1.5(4) + 2y = 20$ $x = 4$

$y = 7$ Solve for y.

The solution is (4, 7).

WatchOut!

Elimination Remember when you add one equation to another to add *every* term, including the constant on the other side of the equal sign.

You can use any of the methods or a combination of the methods for solving systems of equations in two variables to solve systems of equations in three variables.

Example 4 Systems of Equations in Three Variables

Solve the system of equations.

$\mathbf{x - 2y + z = 15}$

$\mathbf{2x + 3y - z = 7}$

$\mathbf{4x + 10y - 5z = -3}$

Step 1 Eliminate one variable by using two pairs of equations.

$x - 2y + z = 15$ Equation 1

$(+)\ [2x + 3y - z = 7]$ Equation 2

$3x + y = 22$ Add.

$5x - 10y + 5z = 75$ $5 \times$ Equation 1

$(+)\ [4x + 10y - 5z = -3]$ Equation 3

$9x = 72$ Add

$x = 8$ Divide.

Step 2 Solve the system of two equations.

$3x + y = 22$ Equation in two variables

$3(8) + y = 22$ $x = 8$

$y = -2$ Solve for y.

Step 3 Substitute the two values into one of the original equations to find z.

$x - 2y + z = 15$ Equation 1

$8 - 2(-2) + z = 15$ $x = 8$ and $y = -2$

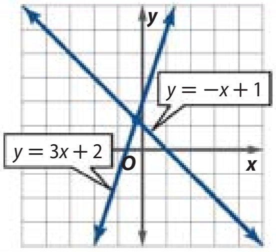
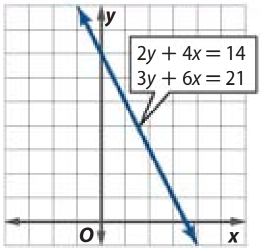
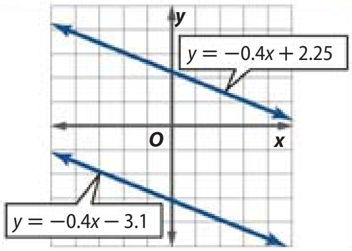
$z = 3$ Solve for z.

The solution is (8, −2, 3).

StudyTip

Other Methods You could have also used the substitution method by first solving for z and then substituting the resulting expression into the other equations.

The graphs of two equations do not always intersect at one point. For example, a system of linear equations could contain parallel lines or the same line. In these cases, the system of equations may have no solution or infinitely many solutions. A **consistent** system has at least one solution. If there is exactly one solution, the system is **independent**. If there are infinitely many solutions, the system is **dependent**. If there is no solution, the system is **inconsistent**.

Consistent and Independent	Consistent and Dependent	Inconsistent
		
$y = 3x + 2$ $y = -x + 1$	$2y + 4x = 14 \rightarrow y = -2x + 7$ $3y + 6x = 21 \rightarrow y = -2x + 7$	$y = -0.4x + 2.25$ $y = -0.4x - 3.1$
different slopes	same slope, same y-intercept	same slope, different y-intercepts
Lines intersect.	Graphs are same line.	Lines are parallel.
one solution	infinitely many solutions	no solution

Example 5 No Solution and Infinitely Many Solutions

Solve each system of equations. State whether the system is *consistent and independent*, *consistent and dependent*, or *inconsistent*.

a. $-7x + 3y = 21$
$7x - 3y = 17$

$$\begin{array}{rl} -7x + 3y = 21 & \text{Add.} \\ (+)\ [7x - 3y = 17] & \\ \hline 0 = 38 & \end{array}$$

Because $0 = 38$ is not true, this system has no solution. Therefore, the system is inconsistent, and the equations in the system are parallel lines, as shown in Figure 0.5.2.

b. $5x + 2y = 12$
$20x + 8y = 48$

$$\begin{array}{rl} 20x + 8y = 48 & 4 \times \text{Equation 1} \\ (-)\ [20x + 8y = 48] & \\ \hline 0 = 0 & \end{array}$$

Because $0 = 0$ is always true, there are an infinite number of solutions. Therefore, the system is consistent and dependent, and the equations in the system have the same graph, as shown in Figure 0.5.3.

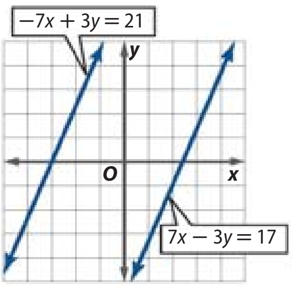


Figure 0.5.2

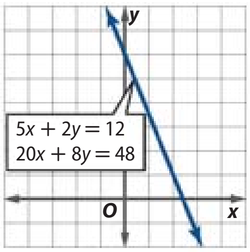


Figure 0.5.3

StudyTip

Consistent Systems Remember that independent and dependent systems are always consistent systems.

2 Systems of Inequalities

Solving a **system of inequalities** means finding all of the ordered pairs that satisfy the inequalities in the system.

KeyConcept Solving Systems of Inequalities

Step 1 Graph each inequality, shading the correct area. Use a solid line to graph inequalities that contain $\geq$ or $\leq$. Use a dashed line to graph inequalities that contain $>$ or $<$.

Step 2 Identify the region that is shaded for all of the inequalities. This is the solution of the system.

Step 3 Check the solution using a test point within the solution region.

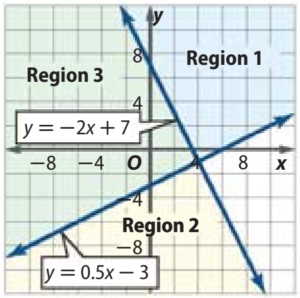
Example 6 Intersecting Regions

Solve the system of inequalities.

$y \geq 0.5x - 3$
$y \leq -2x + 7$

Step 1 Graph each inequality.

Use a solid line to graph each inequality, since each contains either $\geq$ or $\leq$. Then shade the region either above or below the line that contains the coordinates that make each inequality true.



Step 2 Identify the region that is shaded for all of the inequalities.

The solution of $y \geq 0.5x - 3$ is Regions 1 and 3.

The solution of $y \leq -2x + 7$ is Regions 2 and 3.

Region 3 is part of the solution of both inequalities, so it is the solution of the system.

CHECK You can use a test point from the solution region to check your solution. Substitute the x- and y-values of the test point into the inequalities.

$y \geq 0.5x - 3$	$y \leq -2x + 7$
$0 \overset{?}{\geq} 0.5(0) - 3$	$0 \overset{?}{\leq} -2(0) + 7$
$0 \geq -3$ ✓	$0 \leq 7$ ✓

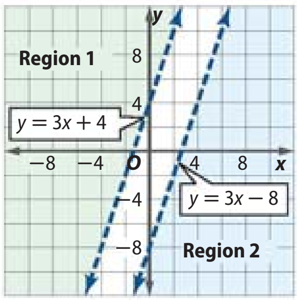
When the regions do not intersect, the system has no solution. That is, the solution set is the empty set.

Example 7 System of Inequalities with Separate Regions

Solve the system of inequalities.

$y < 3x - 8$
$y > 3x + 4$

Graph each line and shade the region either above or below the line that makes the inequality true.



The solution of $y < 3x - 8$ is Region 1.

The solution of $y > 3x + 4$ is Region 2.

Because the graphs of the inequalities do not overlap, there are no points in common and there is no solution to the system.

WatchOut!

Parallel Inequalities Not all systems of inequalities with boundaries having the same slope have no solution. For example, if the system in Example 7 had been
$y > 3x - 8$
$y > 3x + 4$,
the solution would have been the region to the right of the line $y = 3x + 4$.

Exercises

= Step-by-Step Solutions begin on page R29.

Solve each system of equations by graphing. (Example 1)

1. $y = 5x - 2$
   $y = -2x + 5$

2. $y = 2x - 5$
   $y = 0.5x + 1$

3. $x + y = -2$
   $3x - y = 10$

4. $y = -3$
   $2x = 8$

5. $3y = 4x + 6$
   $2y = x - 1$

6. $x = 5$
   $4x + 5y = 20$

Use substitution to solve each system of equations. (Example 2)

7. $5x - y = 16$
   $2x + 3y = 3$

8. $3x - 5y = -8$
   $x + 2y = 1$

9. $y = 6 - x$
   $x = 4.5 + y$

10. $x = 2y - 8$
    $2x - y = -7$

11. $4x - 5y = 6$
    $x + 3 = 2y$

12. $x - 3y = 6$
    $2x + 4y = -2$

13. **JOBS** Connor works at a movie rental store earning \$8 per hour. He also walks dogs for \$10 per hour on the weekends. Connor worked 13 hours this week and made \$110. How many hours did he work at the movie rental store? How many hours did he walk dogs over the weekend?

Use elimination to solve each system of equations. (Example 3)

14. $7x + y = 9$
    $5x - y = 15$

15. $2x - 3y = 1$
    $4x - 5y = 7$

16. $-3x + 10y = 5$
    $2x + 7y = 24$

17. $2x + 3y = 3$
    $12x - 15y = -4$

18. $3x + 4y = -1$
    $6x - 2y = 3$

19. $5x - 6y = 10$
    $-2x + 3y = -7$

Solve each system of equations. (Example 4)

20. $x + 2y + 3z = 5$
    $3x + 2y - 2z = -13$
    $5x + 3y - z = -11$

21. $x - y - z = 7$
    $-x + 2y - 3z = -12$
    $3x - 2y + 7z = 30$

22. $7x + 5y + z = 0$
    $-x + 3y + 2z = 16$
    $x - 6y - z = -18$

23. $3x - 5y + z = 9$
    $x - 3y - 2z = -8$
    $5x - 6y + 3z = 15$

24. $4x + 2y + z = 7$
    $2x + 2y - 4z = -4$
    $x + 3y - 2z = -8$

25. $x - 3z = 7$
    $2x + y - 2z = 11$
    $-x - 2y + 2z = 6$

26. $8x - z = 4$
    $y + z = 5$
    $11x + y = 15$

27. $4x - 2y + z = -5$
    $5x + y + 3z = 6$
    $-2x + 3y + 2z = -4$

Solve each system of equations. State whether the system is *consistent and independent*, *consistent and dependent*, or *inconsistent*. (Example 5)

28. $8x - 5y = -11$
    $-8x + 9y = 7$

29. $x - y = 2$
    $2x = 2y + 10$

30. $5x + 4y = 2$
    $6x + 5y = 4$

31. $12x - 9y = 3$
    $4x - 3y = 1$

32. $1.5x + y = 3.5$
    $3x + 2y = 7$

33. $10x - 3y = -4$
    $-8x + 5y = 11$

34. $2x - 2y + 3z = 2$
    $2x - 3y + 7z = -1$
    $4x - 3y + 2z = 0$

35. $-3x + 2y + z = -23$
    $4x + 2y + z = 5$
    $5x + 3y + 3z = 11$

36. **CAMPING** The Mountaineers Club held two camping trips during the summer. The club rented 5 tents and 1 cabin for the 30 members who went on the first trip. The club rented 4 tents and 2 cabins for the 36 members who went on the second trip. If the tents and cabins were filled to capacity on both trips, how many people can each tent and each cabin accommodate? (Example 5)

Solve each system of inequalities. If the system has no solution, state *no solution*. (Examples 6 and 7)

37. $y \geq x - 3$
    $y \leq 2x + 1$

38. $y + x < 1$
    $y > -x - 1$

39. $x + 2y \geq 12$
    $x - y \geq 3$

40. $y \leq \frac{1}{3}x - 7$
    $3y \geq x + 6$

41. $y + 5 < 4x$
    $2y > -2x + 10$

42. $y \leq -x + 8$
    $y \geq 0.5x - 4$

43. $8y \leq -2x - 1$
    $4y + x \geq 3$

44. $y + 7 < 3x$
    $2y + 5x > 8$

45. $-6y \geq -5x + 6$
    $y \leq -3x - 1$

46. $y + 4 \leq \frac{4}{3}x$
    $3y \geq 4x + 9$

47. $y \leq 2x + 1$
    $y \geq 2x - 2$
    $3x + y \leq 9$

48. $x - 3y > 2$
    $2x - y < 4$
    $2x + 4y \geq -7$

49. **ART** Charlie can spend no more than \$225 on the art club's supply of brushes and paint. He needs at least 20 brushes and 56 tubes of paint. Graph the region that shows how many packages of each item can be purchased. (Example 6)

LESSON 0-6 Matrix Operations

Objective

1. Use characteristics to describe matrices.
2. Add, subtract, and multiply matrices by a scalar.

NewVocabulary

matrix
element
dimensions
row matrix
column matrix
square matrix
zero matrix
equal matrices
scalar

1 Describe and Analyze Matrices

A **matrix** is a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets. Each value in the matrix is called an **element**. A matrix is usually named using an uppercase letter.

The element −1 is in Row 2, Column 1, depicted by a_{21}.

$$A = \underbrace{\begin{bmatrix} 8 & -2 & 5 & 6 \\ -1 & 3 & -3 & 6 \\ 7 & -8 & 1 & 4 \end{bmatrix}}_{\text{4 columns}} \Big\}\text{3 rows}$$

The element −8 is in Row 3, Column 2, depicted by a_{32}.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix, which is read m *by* n. Matrix A above is a 3×4 matrix because it has 3 rows and 4 columns.

Example 1 Dimensions and Elements of a Matrix

Use $A = \begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix}$ to answer the following.

a. State the dimensions of A.

$$\underbrace{\begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix}}_{\text{3 columns}} \Big\}\text{2 rows}$$

Because A has 2 rows and 3 columns, the dimensions of A are 2×3.

b. Find the value of a_{13}.

$$\begin{array}{c} \text{Column 3} \\ \begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix} \end{array} \leftarrow \text{Row 1}$$

Because a_{13} is the element in row 1, column 3, the value of a_{13} is −18.

Certain matrices have special names. For example, a matrix that has one row is called a **row matrix**, and a matrix with one column is a **column matrix**. A matrix that has the same number of rows and columns is known as a **square matrix**, and a matrix in which every element is zero is called a **zero matrix**.

Row Matrix

$$\begin{bmatrix} 8 & -5 & 2 & 4 \end{bmatrix}$$

Column Matrix

$$\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Square Matrix

$$\begin{bmatrix} -4 & 2 \\ 3 & 9 \end{bmatrix}$$

Zero Matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Two matrices are **equal matrices** if and only if each element of one matrix is equal to the corresponding element in the other matrix. So, matrix A and B shown below are equal matrices.

$$A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

Notice that for two matrices to be equal, they must have the same number of rows and columns.

2 Matrix Operations

Matrices can be added or subtracted if and only if they have the same dimensions.

StudyTip

Corresponding Elements *Corresponding* refers to elements that are in the exact same position in each matrix.

KeyConcept Adding and Subtracting Matrices

To add or subtract two matrices with the same dimensions, add or subtract their corresponding elements.

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} + \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{A+B}{\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}}$$

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} - \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{A-B}{\begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}}$$

Example 2 Add and Subtract Matrices

Find each of the following for $A = \begin{bmatrix} 8 & 3 \\ -5 & 14 \end{bmatrix}$, $B = \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix}$, **and** $C = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$.

a. $A + B$

$$A + B = \begin{bmatrix} 8 & 3 \\ -5 & 14 \end{bmatrix} + \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix}$$ Substitution

$$= \begin{bmatrix} 8+12 & 3+(-7) \\ -5+6 & 14+(-23) \end{bmatrix} \text{ or } \begin{bmatrix} 20 & -4 \\ 1 & -9 \end{bmatrix}$$ Add corresponding elements.

b. $B - C$

$$B - C = \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix} - \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$ Substitution

B is a 2×2 matrix and C is a 2×1 matrix. Since these dimensions are not the same, you cannot subtract the matrices.

You can multiply any matrix by a constant called a **scalar**. When you do this, you multiply each individual element by the value of the scalar.

Example 3 Scalar Multiplication

Find each product.

a. $3\begin{bmatrix} -6 & -3 & 7 \\ 10 & 2 & -15 \end{bmatrix}$

$$3\begin{bmatrix} -6 & -3 & 7 \\ 10 & 2 & -15 \end{bmatrix} = \begin{bmatrix} 3(-6) & 3(-3) & 3(7) \\ 3(10) & 3(2) & 3(-15) \end{bmatrix} \text{ or } \begin{bmatrix} -18 & -9 & 21 \\ 30 & 6 & -45 \end{bmatrix}$$

b. $-4\begin{bmatrix} 2 & -9 \\ 7 & 3 \\ -11 & 4 \end{bmatrix}$

$$-4\begin{bmatrix} 2 & -9 \\ 7 & 3 \\ -11 & 4 \end{bmatrix} = \begin{bmatrix} -4(2) & -4(-9) \\ -4(7) & -4(3) \\ -4(-11) & -4(4) \end{bmatrix} \text{ or } \begin{bmatrix} -8 & 36 \\ -28 & -12 \\ 44 & -16 \end{bmatrix}$$

StudyTip

Scalar Multiplication Matrix brackets behave like other grouping symbols. So when multiplying by a scalar, distribute the same way as with a grouping symbol.

Many properties of real numbers also hold true for matrices. A summary of these properties is listed below.

KeyConcept Properties of Matrix Operations

For any matrices A, B, and C for which the matrix sum and product are defined and any scalar k, the following properties are true.

Commutative Property of Addition	$A + B = B + A$
Associative Property of Addition	$(A + B) + C = A + (B + C)$
Left Scalar Distributive Property	$k(A + B) = kA + kB$
Right Scalar Distributive Property	$(A + B)k = kA + kB$

Multi-step operations can be performed on matrices. The order of these operations is the same as with real numbers.

Example 4 Multi-Step Operations

Find $4(P + Q)$ if $P = \begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix}$ and $Q = \begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}$.

$4(P + Q) = 4\left(\begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix} + \begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}\right)$ Substitution

$= 4\begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix} + 4\begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}$ Distributive Property

$= \begin{bmatrix} 12 & 32 & -8 \\ -20 & 20 & -16 \end{bmatrix} + \begin{bmatrix} -16 & 20 & 28 \\ 12 & -40 & -24 \end{bmatrix}$ Multiply by the scalar.

$= \begin{bmatrix} 12 + (-16) & 32 + 20 & -8 + 28 \\ -20 + 12 & 20 + (-40) & -16 + (-24) \end{bmatrix}$ Add.

$= \begin{bmatrix} -4 & 52 & 20 \\ -8 & -20 & -40 \end{bmatrix}$ Simplify.

You can use the same algebraic methods for solving equations with real numbers to solve equations with matrices.

Example 5 Solving a Matrix Equation

Given $A = \begin{bmatrix} -9 & 15 & 4 \\ 2 & -10 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -7 & 8 \\ 14 & 10 & -3 \end{bmatrix}$, solve $4X - B = A$ for X.

$4X - B = A$ Original equation

$4X = A + B$ Add B to each side.

$X = \frac{1}{4}(A + B)$ Divide each side by 4.

$X = \frac{1}{4}\left(\begin{bmatrix} -9 & 15 & 4 \\ 2 & -10 & -5 \end{bmatrix} + \begin{bmatrix} 5 & -7 & 8 \\ 14 & 10 & -3 \end{bmatrix}\right)$ Substitution

$X = \frac{1}{4}\begin{bmatrix} -4 & 8 & 12 \\ 16 & 0 & -8 \end{bmatrix}$ Add.

$X = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & -2 \end{bmatrix}$ Multiply by the scalar.

WatchOut!

Matrix Equations Remember that the variable X in matrix equations stands for a matrix, while the variable x in algebraic equations stands for a number.

Matrix equations can be used in real-world situations.

Real-World Example 6 Use a Matrix Equation

CELL PHONES **Allison took a survey of her high school to see which class sent the most text messages, pictures, and talked for the most minutes on their cell phones each week. The averages for the freshmen, sophomores, juniors, and seniors are shown.**

Class	Texts	Pictures	Calls
freshman	20	3	163
sophomore	25	4	170
junior	15	7	178
senior	22	3	190

Real-WorldLink

On average, 13- to 17-year-olds text more than they talk, sending and receiving over 1,700 text messages a month but only making and receiving about 230 calls per month.

Source: Nielsen Mobile

a. If each text message costs \$0.10, each picture costs \$0.75, and each minute on the phone costs \$0.05, find the average weekly cell phone costs for each class. Express your answer as a matrix.

Step 1 Write a matrix equation for the total cost X. Let T represent the number of texts for all classes, P represent the number of pictures, and C represent the number of call minutes.

$X = 0.10T + 0.75P + 0.05C$

Step 2 Solve the equation.

$X = 0.10T + 0.75P + 0.05C$ Original equation

$$= 0.10\begin{bmatrix}20\\25\\15\\22\end{bmatrix} + 0.75\begin{bmatrix}3\\4\\7\\3\end{bmatrix} + 0.05\begin{bmatrix}163\\170\\178\\190\end{bmatrix}$$ Substitution

$$= \begin{bmatrix}2.00\\2.50\\1.50\\2.20\end{bmatrix} + \begin{bmatrix}2.25\\3.00\\5.25\\2.25\end{bmatrix} + \begin{bmatrix}8.15\\8.50\\8.90\\9.50\end{bmatrix} \text{ or } \begin{bmatrix}12.40\\14.00\\15.65\\13.95\end{bmatrix}$$ Multiply by the scalars.

The final matrix indicates average weekly cell phone costs for each class. Therefore, on average, each freshman spent \$12.40, each sophomore spent \$14.00, each junior spent \$15.65, and each senior spent \$13.95.

b. If there are 100 freshmen, 180 sophomores, 250 juniors, and 300 seniors that use cell phones at Allison's school, use her survey results to estimate the total number of text messages sent, pictures sent, and minutes used on the cell phone each week by these students. Express your answer as a matrix.

Step 1 Write a matrix equation for the total usage X. Let F represent freshmen, S represent sophomores, J represent juniors, and N represent seniors.

$X = 100F + 180S + 250J + 300N$

Step 2 Solve the equation.

$X = 100F + 180S + 250J + 300N$

$= 100[20 \quad 3 \quad 163] + 180[25 \quad 4 \quad 170] + 250[15 \quad 7 \quad 178] + 300[22 \quad 3 \quad 190]$

$= [16{,}850 \quad 3670 \quad 148{,}400]$

The final matrix indicates the average weekly totals for each type of cell phone use. Therefore, there were 16,850 texts, 3670 pictures, and 148,400 minutes used by these students.

Exercises

= Step-by-Step Solutions begin on page R29.

State the dimensions of each matrix. (Example 1)

1. $\begin{bmatrix} 1 & -8 \\ 6 & -2 \end{bmatrix}$

2. $\begin{bmatrix} -9 & -8 \\ 2 & 17 \\ 11 & -6 \end{bmatrix}$

3. $\begin{bmatrix} 10 & 12 & 25 & 48 \\ 53 & 62 & 74 & 89 \end{bmatrix}$

4. $\begin{bmatrix} -5 & -9 & 4 \\ -7 & 12 & 1 \\ 14 & 6 & -8 \end{bmatrix}$

Find the value of each element in

$$A = \begin{bmatrix} -3 & 45 & 28 & -19 \\ 24 & 36 & -22 & 5 \\ 8 & -11 & 54 & 17 \\ -15 & 4 & 29 & -9 \end{bmatrix}.$$ (Example 1)

5. a_{22} **6.** a_{21} **7.** a_{43}

8. a_{13} **9.** a_{32} **10.** a_{34}

Find each of the following for $W = \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix}$,

$X = \begin{bmatrix} 1 & -3 \\ -5 & 9 \\ 12 & 7 \end{bmatrix}$, $Y = \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$, and $Z = \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$.

If the matrix does not exist, write *impossible*. (Example 2)

11. $W + X$ **12.** $Z - X$ **13.** $Z - Y$

14. $X + Y$ **15.** $W - X$ **16.** $Y + Z$

17. **BUSINESS** A car dealership has two used car lots. The matrices below represent the number of cars on each lot by age range i and type of vehicle j. Write a matrix showing the total number of cars of each age range and vehicle type on both lots. (Example 2)

$$[a_{ij}] = \begin{bmatrix} 42 & 56 & 85 \\ 41 & 57 & 89 \\ 45 & 53 & 84 \end{bmatrix} \quad [b_{ij}] = \begin{bmatrix} 51 & 45 & 79 \\ 53 & 48 & 81 \\ 56 & 46 & 83 \end{bmatrix}$$

Find each product. (Example 3)

18. $2\begin{bmatrix} 6 & -18 & 7 \\ 3 & 4 & 11 \end{bmatrix}$

19. $9\begin{bmatrix} -1 & -5 \\ 8 & 4 \end{bmatrix}$

20. $3\begin{bmatrix} 2 & 8 \\ -7 & 15 \\ 12 & -6 \end{bmatrix}$

21. $6[-3 \quad 10 \quad -5 \quad 9]$

22. $7\begin{bmatrix} 20 & -9 & 4 \\ -1 & 5 & 11 \end{bmatrix}$

23. $4\begin{bmatrix} -4 & 6 \\ -12 & 5 \\ 3 & 4 \end{bmatrix}$

24. **SWIMMING** Jessica took her two children to the community swimming pool once a week for six weeks. The daily admission fees are $4.50 for a child and $6.75 for an adult. Write a 1×3 matrix with a scalar multiple that represents the total cost of admission. What is the total cost? (Example 3)

Find each of the following if $D = \begin{bmatrix} -2 & 5 \\ 9 & -11 \\ 4 & -7 \end{bmatrix}$,

$E = \begin{bmatrix} 8 & 10 \\ -5 & 5 \\ 1 & -12 \end{bmatrix}$, and $F = \begin{bmatrix} 5 & -1 \\ -4 & 2 \\ 6 & 10 \end{bmatrix}$. (Example 4)

25. $2D + E$ **26.** $3(E - F)$

27. $\frac{1}{2}(D + F)$ **28.** $3D - 2E$

29. $D + E - F$ **30.** $2(D + F) - E$

Given $J = \begin{bmatrix} 8 & -10 & 3 \\ -4 & 1 & 12 \end{bmatrix}$, $K = \begin{bmatrix} 2 & 5 & -9 \\ -6 & 7 & -3 \end{bmatrix}$ and

$L = \begin{bmatrix} 4 & 1 & -8 \\ 11 & -7 & 6 \end{bmatrix}$, solve each equation for X. (Examples 5 and 6)

31. $2X = J + K$ **32.** $L - K = \frac{1}{3}X$

33 $2J - L = 3X$ **34.** $3K - X = J$

35. $3L - 2K = X$ **36.** $2(J - X) = -L$

Use matrices A, B, C, D, E and F to solve for X. If the matrix does not exist, write *impossible*.

$A = \begin{bmatrix} 5 & 7 \\ -6 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 \\ -1 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 4 & -2 & 3 \\ 5 & 0 & -1 \\ 9 & 0 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 0 \\ 4 & 4 & -2 \end{bmatrix}$ $E = \begin{bmatrix} 8 & -4 & 2 \\ 3 & 1 & -5 \end{bmatrix}$ $F = \begin{bmatrix} -6 & -1 & 0 \\ 1 & 4 & 0 \end{bmatrix}$

37. $A + B = X$ **38.** $X = -2F$

39. $C - D = X$ **40.** $X = D + B$

41. $X = 4D$ **42.** $X = 3B - A$

43. $F - 2(E + C) = X$ **44.** $2X = 3(E + F)$

45. **SWIMMING** The table shows some of the women's freestyle swimming records.

Distance (meters)	World	Olympic	American
50	23.96 s	24.06 s	24.07 s
100	52.88 s	53.12 s	53.39 s
200	1 min 54.47 s	1 min 54.82 s	1 min 55.78 s
800	8 min 14.10 s	8 min 14.10 s	8 min 16.22 s

Source: Fédération Internationale de Natation

a. Find the difference between American and World records expressed as a column matrix.

b. If all the data in the table were expressed in seconds and represented by a matrix A, what matrix expression could be used to convert all the data to minutes?

LESSON 0-7

Probability with Permutations and Combinations

Objective

1 Find the number of possible outcomes of an experiment.

2 Use permutations and combinations with probability.

NewVocabulary

experiment
sample space
independent events
dependent events
factorial
permutation
combination

1 Sample Space An **experiment** is a situation involving chance or probability that leads to specific outcomes. The set of all possible outcomes is called the **sample space**. One method that can be used to determine the *number* of possible outcomes of an experiment is the Fundamental Counting Principle.

KeyConcept Fundamental Counting Principle

Let A and B be two events. If event A has n_1 possible outcomes and is followed by event B that has n_2 possible outcomes, then event A followed by event B has $n_1 \cdot n_2$ possible outcomes.

The Fundamental Counting Principle can also be used to find the number of possible outcomes for three or more events. For example, the number of ways that k events can occur is given by $n_1 \cdot n_2 \cdot n_3 \cdot \cdots \cdot n_k$.

Events with outcomes that do not affect each other are called **independent events**, and events with outcomes that do affect each other are called **dependent events.**

Example 1 Fundamental Counting Principle

a. A restaurant offers a dinner special in which a customer can select from one of 6 appetizers, a soup or salad, one of 12 entrees, and one of 8 desserts. How many different dinner specials are possible?

Because the selection of one menu item does not affect the selection of any other item, each selection is independent. To determine the number of possible dinner specials, multiply the number of ways each item can be selected.

$6 \cdot 2 \cdot 12 \cdot 8 = 1152$

Therefore, there are 1152 different dinner specials.

b. Garrett works for a bookstore. He is arranging the five best-sellers for a shelf display. If he can place the books in any order, how many different ways can Garrett arrange the books?

The selection of the book for the first position affects the books available for the second position, the selection for the second position affects the books available for the third position, and so on. So, the selections of books are dependent events.

There are 5 books from which to choose for the first position, 4 books for the second, 3 for the third, 2 for the fourth, and 1 for the fifth. To determine the total number of ways that the books can be arranged, multiply by the number of ways that the books can be chosen for each position.

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Therefore, there are 120 possible ways for Garrett to arrange the books.

The expression used in Example 1b to calculate the number of arrangements of books, $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, can be written as 5!, which is read *5 factorial*. The **factorial** of a positive integer n is the product of the positive integers less than or equal to n, and is given by

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 1, \text{ where } 0! = 1.$$

2 Permutations and Combinations

The Fundamental Counting Principle can also be used to determine the number of ways that n objects can be arranged in a certain order. An arrangement of n objects is called a **permutation** of the objects.

KeyConcept Permutations

The number of permutations of n objects taken n at a time is

$${}_nP_n = n!.$$

The number of permutations of n objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!}.$$

Example 2 Permutations with Probability

An alarm system requires a 7-digit code using the digits 0 through 9. Each digit may be used only once.

a. How many different codes are possible?

The order of the numbers in the code is important, so this situation is a permutation of 10 digits taken 7 at a time.

$${}_nP_r = \frac{n!}{(n-r)!} \qquad \text{Definition of a permutation}$$

$${}_{10}P_7 = \frac{10!}{(10-7)!} \qquad n = 10 \text{ and } r = 7$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} \qquad \text{Expand 10! and divide out common factorials.}$$

$$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \qquad \text{Simplify.}$$

$$= 604{,}800 \qquad \text{Multiply.}$$

So, 604,800 codes are possible.

StudyTip

Arrangements In a permutation, the order of the objects is important. For example, when arranging two objects A and B using a permutation, the arrangement AB is different from the arrangement BA.

b. If a code is randomly generated, what is the probability that the first three digits are odd?

To find the probability of the first three digits being odd, find the number of ways to select three odd digits and multiply by the number of ways to select the remaining digits and then divide by the total possible codes.

$$P(\text{1st three digits are odd}) = \frac{\text{ways to select 3 odd digits} \cdot \text{ways to select last 4 digits}}{\text{total possible codes}}$$

$$= \frac{{}_5P_3 \cdot {}_7P_4}{{}_{10}P_7}$$

$$= \frac{\frac{5!}{(5-3)!} \cdot \frac{7!}{(7-4)!}}{\frac{10!}{(10-7)!}} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{\frac{5!}{2!} \cdot \frac{7!}{3!}}{\frac{10!}{3!}} \qquad \text{Subtract.}$$

$$= \frac{\frac{5 \cdot 4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}}}{\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}}} \qquad \text{Expand 5!, 7!, and 10!, and divide out common factorials.}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4} \qquad \text{Simplify.}$$

$$= \frac{50{,}400}{604{,}800} \text{ or } \frac{1}{12} \qquad \text{Multiply.}$$

Therefore, the probability is $\frac{1}{12}$ or about 0.08.

In a combination, order is *not* important. A **combination** of n objects taken r at a time is calculated by dividing the number of permutations by the number of arrangements containing the same elements and is denoted by ${}_nC_r$.

KeyConcept Combinations

The number of combinations of n objects taken r at a time is

$${}_nC_r = \frac{n!}{(n-r)!\,r!}.$$

The main difference between a permutation and a combination is whether order is considered (as in permutation) or not (as in combination). For example, for objects E, F, G, and H taken two at a time, the permutations and combinations are listed below.

WatchOut

Combinations Not all everyday uses of the word *combination* are descriptions of mathematical combinations. For example, the combination to a lock is described by a permutation.

Permutations			
EF	FE	GE	HE
EG	FG	GF	HF
EH	FH	GH	HG

Combinations	
EF	FG
EG	FH
EH	GH

In permutations, EF is different from FE. But in combinations, EF is the same as FE.

Example 3 Combinations with Probability

There are 7 seniors, 5 juniors, and 4 sophomores on the pep squad. Mr. Rinehart needs to choose 12 students out of the group to sell spirit buttons during lunch.

a. How many ways can the 12 students be chosen?

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$ Definition of combination

$${}_{16}C_{12} = \frac{16!}{(16-12)!\,12!}$$ $n = 16$ and $r = 12$

$$= \frac{16!}{4!\,12!}$$ Subtract.

$$= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{12!}}$$ Expand 16! and 4!, and divide out common factorials.

$$= \frac{43{,}680}{24}$$ Multiply.

$$= 1820$$ Simplify.

So, there are 1820 ways that the 12 students can be chosen.

b. If the students are randomly chosen, what is the probability that 4 seniors, 4 juniors, and 4 sophomores will be chosen?

ways to choose 4 seniors out of 7: ${}_7C_4 = \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!}$

ways to choose 4 juniors out of 5: ${}_5C_4 = \frac{5!}{(5-4)!\,4!} = \frac{5!}{1!\,4!}$

ways to choose 4 sophomores out of 4: ${}_4C_4 = \frac{4!}{(4-4)!\,4!} = \frac{4!}{0!\,4!}$

There are $\frac{7!}{3!\,4!} \cdot \frac{5!}{1!\,4!} \cdot \frac{4!}{0!\,4!}$ or 175 ways to choose 4 seniors, 4 juniors, and 4 sophomores.

Therefore, the probability is $\frac{175}{1820}$ or $\frac{5}{52}$.

Real-WorldLink

Two hundred thousand people from around the world volunteered to be part of the pep squad for the 2008 Summer Olympics. Of this number, about 600 made the team.

Source: *NFL FanhouseRecords*

JUNG YEON-JE/AFP/Getty Images

Exercises

= Step-by-Step Solutions begin on page R29.

Use the Fundamental Counting Principle to determine the number of outcomes for each event. (Example 1)

1. How many different T-shirts are available?

Size	Colors
XS, S, M, L, XL, XXL	blue, red, green, gray, black

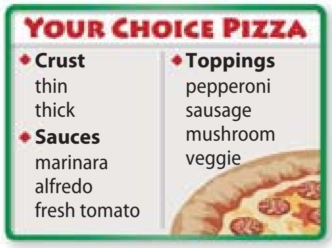
2. For a particular model of car, a dealer offers 3 sizes of engines, 2 types of stereos, 18 body colors, and 7 upholstery colors. How many different possibilities are available for that model?

3. If you toss a coin, roll a die, and then spin a 4-colored spinner with equal sections, how many outcomes are possible?

4. If a deli offers 12 different meats, 5 different cheeses, and 6 different breads, how many different sandwiches can be made with 1 type of meat, 1 type of cheese, and 1 type of bread?

5. An ice cream shop offers 20 flavors of ice cream, 5 different toppings, and 3 different sizes. How many different sundaes are available?

6. How many different 1-topping pizzas are available?



7. How many ways can six different books be arranged on a shelf if the books can be arranged in any order?

8. How many ways can eight actors be listed in the opening credits of a movie if the leading actor must be listed first?

Find each value. (Examples 2 and 3)

9. ${}_6P_6$ **10.** ${}_5P_3$ **11.** ${}_7C_4$

12. ${}_{20}C_{15}$ **13.** ${}_8P_1$ **14.** ${}_6P_4$

15. ${}_6P_3$ **16.** ${}_7P_4$ **17.** ${}_9P_5$

18. ${}_4C_2$ **19.** ${}_{12}C_4$ **20.** ${}_9C_9$

21. CLASS OFFICERS At Grant Senior High School, there are 15 names on the ballot for junior class officers. Five will be selected to form a class committee. (Examples 2 and 3)

a. How many different committees can be formed?

b. In how many ways can the committee be formed if each student has a different responsibility?

c. If there are 8 girls and 7 boys on the ballot, what is the probability that a committee of 2 boys and 3 girls is formed?

22. ART An art gallery curator wants to select four paintings out of twenty to put on display. How many groups of four paintings can be chosen?

23. PHONE NUMBERS In the United States, standard local telephone numbers consist of 7 digits, where the first digit cannot be 1 or 0.

a. Find the number of possibilities for telephone numbers.

b. Find the probability of randomly selecting a given telephone number from all the possible numbers.

c. How many different telephone numbers are possible if only even digits are used?

d. Find the probability of choosing a telephone number in which only even digits are used.

e. Find the number of possibilities for telephone numbers if the first three digits are 593. What is the probability of randomly choosing a telephone number in which the first three digits are 593?

24. CARDS Five cards are drawn from a standard deck of 52 cards.

a. Determine the number of possible five-card selections.

b. Find the probability of an arrangement containing 3 hearts and 2 clubs.

c. Find the probability of an arrangement containing all face cards.

d. Find the probability of an arrangement containing 1 ace, 2 jacks, and 2 kings.

A gumball machine contains 7 red (R), 8 orange (N), 9 purple (P), 7 white (W), and 5 yellow (Y) gumballs. Tyson buys 3 gumballs. Find each probability, assuming that the machine dispenses 3 gumballs at random all at once.

25. P(3 R) **26.** P(2 W and 1 P)

27 P(1 R and 2 N) **28.** P(1 N and 2 Y)

29. P(2 R and 1 Y) **30.** P(1 P, 1 W, and 1 R)

31. COMPUTERS A circuit board with 20 computer chips contains 4 chips that are defective. If 3 chips are selected at random, what is the probability that all 3 are defective?

32. BOOKS Dan has twelve books on his shelf that he has not read, including seven novels and five biographies. If he wants to take four books with him on vacation, what is the probability that he randomly selects two novels and two biographies?

33. SCHOLARSHIPS Twelve male and 16 female students have been selected as equal qualifiers for 6 college scholarships. If the awarded recipients are to be chosen at random, what is the probability that 3 will be male and 3 will be female?

LESSON 0-8 Statistics

Objective

1. Find measures of center and spread.
2. Organize statistical data.

NewVocabulary

statistics
univariate data
measure of central tendency
population
sample
mean, median, mode
measures of spread (or variation)
range
variance
standard deviation
frequency distribution
class (or interval)
relative frequency
class width
cumulative frequency
cumulative relative frequency
quartiles
five-number summary
interquartile range
outliers

1 Measures of Center and Spread **Statistics** is the science of collecting, analyzing, interpreting, and presenting data. The branch of statistics that focuses on collecting, summarizing, and displaying data is known as *descriptive statistics*. Data in one *variable*, or data type, are called **univariate data**. These data can be described by a **measure of central tendency** which represents the center or middle of the data. The three most common measures of central tendency are mean, median, and mode.

A **population** is the entire membership of people, objects, or events of interest to be analyzed. A **sample** is a subset of a population. The formulas for mean use x to represent the data values in a sample or population, Σx to represent the sum of all x-values, n to represent the number of x-values, μ to represent the population mean, $\bar{x}$ to represent the sample mean.

KeyConcept Measures of Central Tendency

Mean the sum of the numbers in a set of data divided by the number of items

Population Mean: $\mu = \dfrac{\Sigma x}{n}$ Sample Mean: $\bar{x} = \dfrac{\Sigma x}{n}$

Median the middle number in a set of data when the data are arranged in numerical order or the mean of the middle two values

Mode the number or numbers that appear most often in a set of data

Example 1 Find Measures of Central Tendency

Find the mean, median, and mode for the data 14, 7, 12, 4, 13, 20, 2, 3, 5, 15, 10, 4.

Mean $\dfrac{\Sigma x}{n} = \dfrac{14 + 7 + 12 + 4 + 13 + 20 + 2 + 3 + 5 + 15 + 10 + 4}{12} \approx 9.08$

Median 2 3 4 4 5 7 10 12 13 14 15 20

$\dfrac{7 + 10}{2}$ or 8.5

Mode The value that occurs most often in the set is 4, so the mode is 4.

Measures of spread or **variation** describe the distribution of a set of data. Three measures of spread are range, variance, and standard deviation. The formulas for population variance σ^2 and standard deviation σ use $x - \mu$ to represent the deviation or *difference* an x-value is from the population mean and $\Sigma(x - \mu)^2$ to represent the sum of the squares of these deviations. Similar notation is used for sample variance s^2 and standard deviation s.

KeyConcept Measures of Spread

Range the difference between the greatest and least values in a set of data

Variance the mean of the squares of the deviations from the mean

Population Variance: $\sigma^2 = \dfrac{\Sigma(x - \mu)^2}{n}$ Sample Variance: $s^2 = \dfrac{\Sigma(x - \bar{x})^2}{n - 1}$

Standard Deviation the average amount by which individual items deviate from the mean of all the data

Population Standard Deviation: $\sigma = \sqrt{\dfrac{\Sigma(x - \mu)^2}{n}}$ Sample Standard Deviation: $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n - 1}}$

StudyTip

Population vs. Sample Statisticians have found that using sample data to approximate measures of spread for a population consistently underestimates these measures. To counteract this error, the formulas for sample variance and standard deviation use division by $n - 1$ instead of n.

Example 2 Find Measures of Spread

> **StudyTip**
>
> **Rounding** The mean and standard deviation should be rounded to one more decimal place than the original data. To avoid rounding error, round only final answers. Avoid using a rounded value to do further calculations.

The quiz scores for a class of 25 students are shown.

Quiz Scores				
7	8	9	9	9
10	5	7	5	6
10	9	7	8	2
8	9	9	7	5
3	6	8	10	10

a. Find the measures of spread for the entire class.

Range maximum − minimum

$= 10 - 2$ or 8

Variance Find the mean of the data.

$\mu = \dfrac{\Sigma x}{n}$ Mean of a population

$= \dfrac{7 + 8 + \ldots + 10 + 10}{25}$ Σx is the sum of the data values and $n = 25$.

$= 7.44$ or about 7.4 Simplify.

Use the unrounded mean to find the variance.

$\sigma^2 = \dfrac{\Sigma(x - \mu)^2}{n}$ Variance of a population

$= \dfrac{\Sigma(x - 7.44)^2}{25}$ $\mu = 7.4$ and $n = 25$

$= \dfrac{(7 - 7.44)^2 + (8 - 7.44)^2 + \ldots + (10 - 7.44)^2}{25}$ Substitution

$= 4.5664$ or about 4.6 Simplify.

Standard Deviation Take the square root of the variance.

$\sigma = \sqrt{4.5664}$

≈ 2.1

b. Use the last column of the quiz scores to find the measures of spread for a sample of the class.

Range The sample is 9, 6, 2, 5, and 10. The range of the sample is $10 - 2$ or 8.

Variance The sample mean is $\bar{x} = \dfrac{9 + 6 + 2 + 5 + 10}{5}$ or 6.4.

$s^2 = \dfrac{\Sigma(x - \bar{x})^2}{n - 1}$ Variance of a sample

$= \dfrac{\Sigma(x - 6.4)^2}{5 - 1}$ $\bar{x} = 6.4$ and $n = 5$

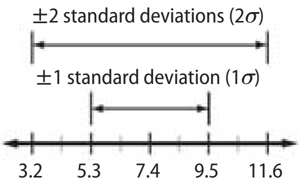
$= \dfrac{(9 - 6.4)^2 + (6 - 6.4)^2 + (2 - 6.4)^2 + (5 - 6.4)^2 + (10 - 6.4)^2}{4}$ Substitution

$= 10.3$ Simplify.

Standard Deviation $\sigma = \sqrt{10.3}$

≈ 3.2

In a given set of data, the majority of the values fall within one standard deviation of the mean, and almost all of the values will fall within 2 standard deviations. The quiz scores in Example 2a had a mean of about 7.4 and a standard deviation of about 2.1. This can be illustrated graphically.



If the quiz scores were compared with other scores throughout the country on a national test, this class would be considered a sample of all of the students who took the test. A sample mean $\bar{x}$ and a sample standard deviation s need to be calculated.

When comparing data sets, it is important to analyze the center *and* spread of each distribution. This is important because two sets of data can have the same mean but different spreads.

Example 3 Compare Data Sets Using Measures of Spread

HEALTH ***Metabolic rate*** **is the rate at which the body consumes energy, measured in Calories per 24 hours. During a study on diet and exercise, the metabolic rates for two different groups of men were observed. Which group has a greater variation in metabolic rates?**

Group 1				
1507	1619	1731	1468	1533
1744	1588	1675	1552	1475
1593	1745	1523	1590	1764
1429	1604	1574	1708	1656

Group 2				
1498	1589	1634	1702	1629
1621	1629	1589	1592	1603
1573	1476	1613	1585	1582
1723	1619	1615	1601	1607

Enter the data into L1 and L2 on a graphing calculator. Press STAT and select 1-Var Stats from the CALC menu, press 2nd [L1] or 2nd [L2] to select the Group 1 or Group 2 data, and press ENTER. Record the values for the sample mean $\bar{x}$, median Med, standard deviation Sx, and use maxX − minX to calculate the range.

Group 1 mean = 1603.9
range = 335
standard deviation = 100.2
median = 1691.5 − 1429 or 1591.5

Group 2 mean = 1604
range = 247
standard deviation = 54.6
median = 1723 − 1476 or 1605

Although the measures of center are reasonably close, the standard deviation of 100.2 for Group 1 is much larger than Group 2's value of 54.6. The range for Group 1 is also much larger than that of Group 2. Therefore, there is a greater variation in metabolic rates in Group 1.

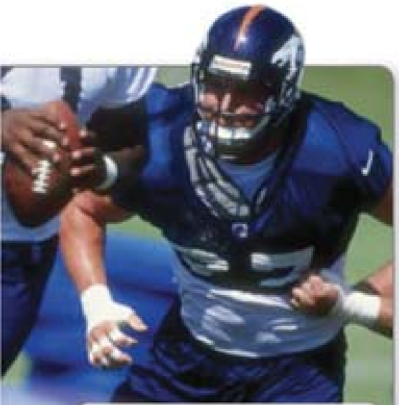
StudyTip

Class Boundaries When data values can be non-integer values such as 19.2, *class boundaries* are used to avoid gaps in data. Class boundaries should have one additional place value than the class limit and end in a 5. In Example 4, the class boundaries for the first and second class limits would be 9.5 to 19.5 and 19.5 to 29.5.

2 Organize Data

Data can be organized into a table called a **frequency distribution** to show how often each data value or group of data values, called a **class** or **interval**, appears in a data set. The **relative frequency** of a class is the ratio of data within the class to all the data. The **cumulative frequency** for a class is the sum of its frequency and all previous classes. The **cumulative relative frequency** for a class is the ratio of the cumulative frequency of the class to all the data.

Each class can be described in several ways. The **class width** is the range of values for each class. A *lower class limit* is the least value that can belong to a specific class, and an *upper class limit* is the greatest value that can belong to a specific class.



Real-WorldLink

Mike Lodish has played in more Super Bowls than anyone else, four times with the Buffalo Bills and twice with the Denver Broncos.

Source: About: Football

Real-World Example 4 Frequency Distribution

FOOTBALL **The winning scores for the first 42 Super Bowls are shown below.**
35 33 16 23 16 24 14 24 16 21 32 27 35 31 27 26 27 38 38 46 39
42 20 55 20 37 52 30 49 27 35 31 34 23 34 20 48 32 24 21 29 17

a. Make a distribution table that shows the frequency and relative frequency of the data.

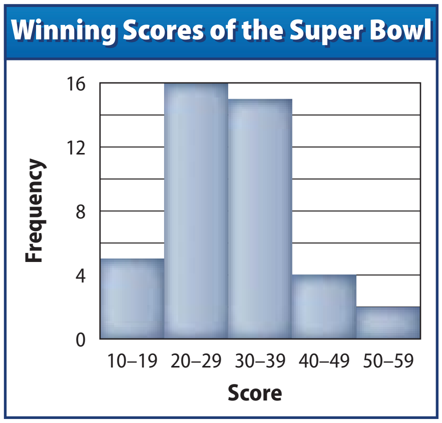
Step 1 Determine the number of classes and an appropriate class interval. The scores range from 14 to 55, so use 5 classes with a class interval of 10 points. Make a table listing the class limits. Begin with 10 points and end with 59 points.

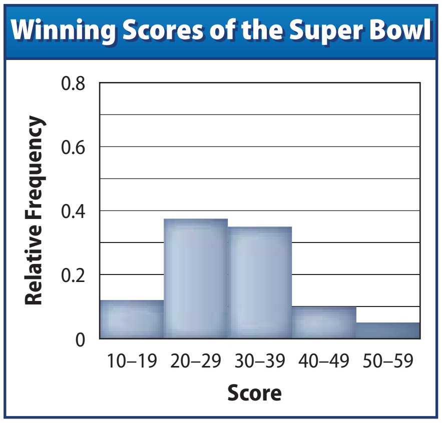
Step 2 Tally the data. Then calculate the relative frequencies.

Winning Score	Tallies	Frequency	Relative Frequency
10–19	𝍸	5	$\frac{5}{42}$ or about 0.12
20–29	𝍸 𝍸 𝍸 |	16	$\frac{16}{42}$ or about 0.38
30–39	𝍸 𝍸 𝍸	15	$\frac{15}{42}$ or about 0.36
40–49	||||	4	$\frac{4}{42}$ or about 0.10
50–59	||	2	$\frac{2}{42}$ or about 0.05
		42	

Brian Bahr/Getty Images Sport/Getty Images

b. Construct a histogram for both the frequency distribution and the relative frequency distribution. Then compare the graphs.





The overall shapes of the histograms are the same. The only difference is the vertical scale.

Study Tip

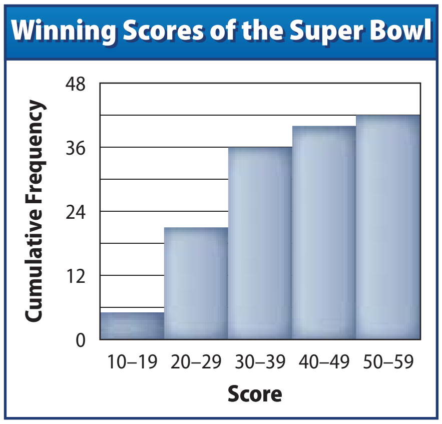
Cumulative Frequencies The cumulative frequency for the last class should always equal the total number of data values. Likewise, the cumulative relative frequency should always equal 1.

c. Make a cumulative frequency distribution for the data. Then determine the cumulative relative frequency distribution.

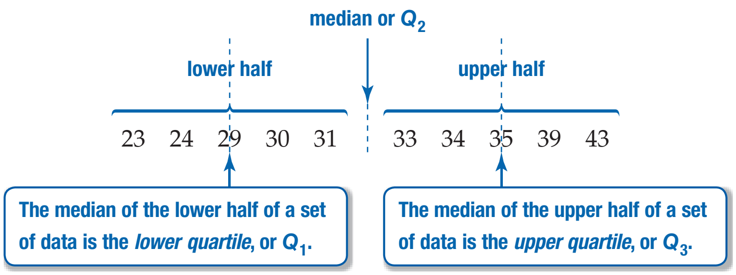
Winning Score	Frequency	Cumulative Frequency	Cumulative Relative Frequency
10–19	5	5	$\frac{5}{42}$ or about 0.12
20–29	16	16 + 5 or 21	$\frac{21}{42}$ or 0.50
30–39	15	15 + 21 or 36	$\frac{36}{42}$ or about 0.86
40–49	4	4 + 36 or 40	$\frac{40}{42}$ or about 0.95
50–59	2	2 + 40 or 42	$\frac{42}{42}$ or 1

d. Construct a histogram for the cumulative frequency distribution. Then compare the graph to the graph of the frequency distribution.

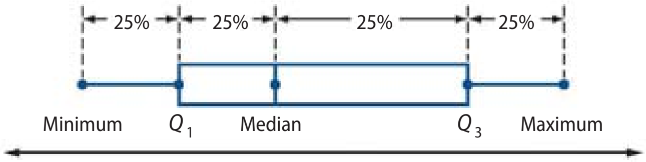
The shape of the cumulative frequency histogram shows an increasing pattern. There is a large increase in the number of teams scoring 10 to 19 points to the number of teams scoring 20 to 39 points. Then there is very little change in the number of teams scoring 40 or more points.



In a set of data, **quartiles** are values that divide the data into four equal parts.



This **five-number summary**, which includes the minimum value, lower quartile, median, upper quartile, and the maximum value of a data set, provides another numerical way of characterizing a set of data. The five-number summary can be described visually with a box-and-whisker plot, as shown.



StudyTip

Box-and-Whisker Plot Notice that the box in a box-and-whisker plot represents the middle 50% of the data, while the whiskers represent the upper and lower 25% of the data.

The difference between the upper quartile and lower quartile is called the **interquartile range**. **Outliers** are data that are more than 1.5 times the interquartile range beyond the upper or lower quartiles.

Real-World Example 5 Box-and-Whisker Plots

EDUCATION **The enrollment for state universities in Ohio is shown. Display the data using a box-and-whisker plot.**

greatest value

upper quartile $\frac{23{,}007 + 22{,}819}{2}$ or 22,913

median

lower quartile $\frac{15{,}038 + 13{,}595}{2}$ or 14,316.5

least value

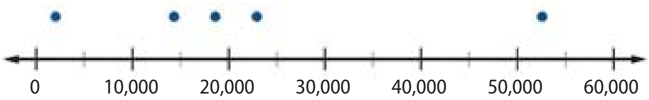
College	Enrollment Fall 2007
The Ohio State University	52,568
University of Cincinnati	29,315
University of Akron	23,007
Kent State University	22,819
Ohio University	21,089
University of Toledo	19,767
Bowling Green State University	18,619
Wright State University	16,151
Miami University	15,968
Cleveland State University	15,038
Youngstown State University	13,595
Shawnee State University	3699
Central State University	2022

Source: National Center for Educational Statistics

StudyTip

Ordering Data In Example 5, notice that the data in the table is given in descending order. If a data set is not listed in ascending or descending order, be sure to order the data before finding the median, upper quartile, and lower quartile values.

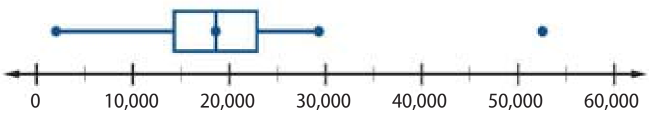
Step 1 Find the maximum and minimum values, and draw a number line that covers the range of the data. Then find the median and the upper and lower quartiles. Mark these points and the extreme values above the number line.



Step 2 Identify any outliers.

The interquartile range is 22,913 − 14,316.5 or 8596.5. There are no data values less than 14,316.5 − 1.5(8596.5) or 1421.75. There is one data value greater than 22,913 + 1.5(8596.5) or 35,807.75. The enrollment at The Ohio State University, 52,568, is an outlier.

Step 3 Draw a box around the upper and lower quartiles, a vertical line through the median, and use horizontal lines or *whiskers* to connect the lower value and the greatest value that is not an outlier. The greatest value that is not an outlier is 29,315.



Exercises

● = Step-by-Step Solutions begin on page R29.

Find the mean, median, and mode of each set of data. (Example 1)

1. {24, 28, 21, 37, 31, 29, 23, 22, 34, 31}

2. {64, 87, 62, 87, 63, 98, 76, 54, 87, 58, 70, 76}

3. {6, 9, 11, 11, 12, 7, 6, 11, 5, 8, 10, 6}

4. **PACKING** Crates of books are being stored. The weights of the crates in pounds are shown in the table. (Example 1)

Weights in Pounds				
142.6	160.8	151.3	139.1	145.2
117.9	172.4	155.7	124.5	126.4
133.8	141.6	119.4	121.2	157.0

   a. What is the mean of the weights?

   b. Find the median of the weights.

   c. If 5 pounds is added to each crate, how will the mean and median be affected?

Find the range, variance, and standard deviation for each set of data. Use the formula for a population. (Example 2)

5. {\$4.45, \$5.50, \$5.50, \$6.30, \$7.80, \$11.00, \$12.20, \$17.20}

6. {200, 476, 721, 579, 152, 158}

7. {5.7, 5.7, 5.6, 5.5, 5.3, 4.9, 4.4, 4.0, 4.0, 3.8}

8. {369, 398, 381, 392, 406, 413, 376, 454, 420, 385, 402, 446}

9. **HEIGHTS** The heights in inches of Ms. Turner's astronomy students are listed below. (Example 2)

Heights in Inches									
66	72	70	74	64	65	60	62	66	67
68	71	70	72	73	65	63	62	62	61

   a. Find the measures of spread for the heights of the astronomy students.

   b. Using the second row as a sample, find the measures of spread for the sample of the astronomy students.

10. **MANUFACTURING** Sample lifetimes, measured in number of charging cycles, for two brands of rechargeable batteries are shown. Which brand has a greater variation in lifetimes? (Example 3)

Brand A				
998	950	1020	1003	990
942	1115	973	1018	981
1047	1002	997	1110	1003

Brand B				
892	1044	1001	999	903
950	998	993	1002	995
990	1000	1005	997	1004

11. **GRAPHIC NOVELS** The prices of 18 randomly selected graphic novels at two stores are shown. Which store has a greater variation in graphic novel prices? (Example 3)

Store 1 (\$)					
18.99	12.99	15.95	12.99	12.95	29.95
24.99	39.99	9.95	14.99	24.95	9.99
17.99	13.99	4.99	29.95	9.99	12.95

Store 2 (\$)					
19.99	7.95	7.95	4.99	12.99	7.95
25.65	7.95	9.99	14.99	9.95	14.99
8.99	9.99	7.95	12.95	14.95	29.95

12. **HISTORY** The ages of the first 44 presidents upon taking office are listed below. (Example 4)

Ages of U.S. Presidents										
57	61	57	57	58	57	61	54	68	51	49
64	50	48	65	52	56	46	54	49	50	47
55	55	54	42	51	56	55	51	54	51	60
62	43	55	56	61	52	69	64	46	54	47

   a. Make a distribution table of the data. Show the frequency, relative frequency, cumulative frequency, and cumulative relative frequency.

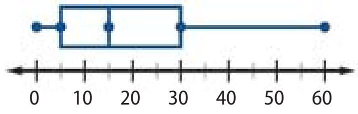
   b. Construct histograms for the frequency, relative frequency, and cumulative relative frequency distributions.

   c. Name the interval or intervals that describe the age of most presidents upon taking office.

13. **SPORTS DRINKS** Carlos surveyed his friends to find the number of bottles of sports drinks that they consume in an average week. Display the data using a box-and-whisker plot. (Example 5)

Sports Drinks Consumed Weekly										
0	0	0	1	1	1	2	2	3	4	4
5	5	7	10	10	10	11	11			

14. **DRIVING** Kara surveyed 20 randomly selected students at her school about how many miles they drive in an average day. The results are shown. (Example 5)



   a. What percent of the students drive more than 30 miles in a day?

   b. What is the interquartile range of the box-and-whisker plot shown?

   c. Does a student at Kara's school have a better chance of meeting someone who drives about the same mileage that he or she does if 15 miles or 50 miles are driven in a day? Why?

CHAPTER 0 Posttest

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*.

1. M is the set of natural number multiples of 5 that are less than 50. $12 \in M$

2. S is the set of integers that are less than -40 but greater than -50. $-49 \in S$

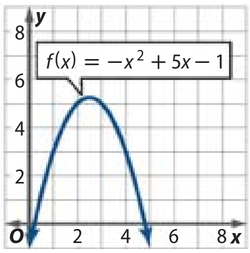
Let $B = \{0, 1, 2, 3\}$, $C = \{0, 1, 2, 3, 4, 5, 6\}$, $D = \{1, 3, 5, 7, 9\}$, $E = \{0, 2, 4, 6, 8, 10\}$, and $F = \{0, 10\}$. Find each of the following.

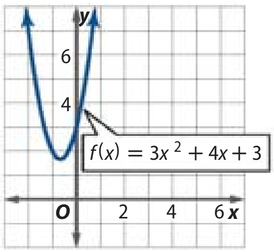
3. $D \cap C$
4. $D \cap F$
5. $E \cup B$
6. $D \cup F$

Simplify.

7. $(1 + 4\boldsymbol{i}) + (-2 - 3\boldsymbol{i})$
8. $(2 + 4\boldsymbol{i}) - (-1 + 5\boldsymbol{i})$
9. $(6 + 7\boldsymbol{i})(-5 + 3\boldsymbol{i})$
10. $(-1 + \boldsymbol{i})(-6 + 2\boldsymbol{i})$
11. $\dfrac{2 + 3\boldsymbol{i}}{1 - 3\boldsymbol{i}}$
12. $\dfrac{1 + 2\boldsymbol{i}}{1 - 2\boldsymbol{i}}$

Determine whether each function has a *maximum* or *minimum* value. Then find the value of the maximum or minimum, and state the domain and range of the function.

13. 

14. 

Solve each equation.

15. $x^2 - x - 72 = 0$
16. $x^2 - 6x + 4 = 0$
17. $2x^2 - 5x + 4 = 0$
18. $2x^2 - x - 3 = 0$

19. **RECREATION** The current value C and the original value v of a recreational vehicle are related by $C = v(1 - r)^n$, where r is the rate of depreciation per year and n is the number of years. If the current value of a recreational vehicle is \$47,500, what would be the value of the vehicle after 75 months at an annual depreciation rate of 15%?

Simplify each expression.

20. $\sqrt[6]{x^{18}y^{20}}$
21. $\sqrt[5]{a^{10}b^7}$
22. $\sqrt{16t^8u^{16}}$
23. $\sqrt[5]{243x^{10}y^{25}z^6}$

Simplify.

24. $\dfrac{y^{\frac{3}{4}} \cdot y^{\frac{2}{3}}}{y^{\frac{5}{12}}}$
25. $\sqrt[9]{512x^{10}y^{28}}$
26. $\sqrt[4]{m^{21}n^{18}}$
27. $\dfrac{\sqrt[12]{25}}{\sqrt[9]{125}}$

28. **JOBS** Leah babysits during the day for \$10 per hour and at night for \$15 per hour. If she worked 5 hours and earned \$60, how many hours did she babysit during the day? How many at night?

Solve each system of equations. State whether the system is *consistent and independent*, *consistent and dependent*, or *inconsistent*.

29. $9x - 4.5y = 15$
$6x - 3y = 10$

30. $5x + y = 2$
$x - y = 22$

31. $9x - 3y + 12z = 39$
$12x - 4y + 16z = 54$
$3x - 8y + 12z = 23$

32. $6x + 2y + 4z = 2$
$3x + 4y - 8z = -3$
$-3x - 6y + 12z = 5$

Solve each system of inequalities. If the system has no solution, state *no solution*.

33. $y \geq x - 3$
$y \leq 3x + 1$

34. $y + x < 6$
$y > -3x + 2$

35. $3x + 2y \geq 6$
$4x - y \geq 2$

36. $2x + 5y \leq -15$
$y > -\frac{2}{5}x + 2$

Find each of the following for $A = \begin{bmatrix} 3 & 0 \\ 2 & -1 \\ -6 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -7 \\ -8 & 4 \\ 10 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 2 \\ -7 & 8 \\ -1 & -3 \end{bmatrix}$.

37. $A + B + C$
38. $B - C$
39. $2A - B$

Find each permutation or combination.

40. ${}_{10}C_3$
41. ${}_{10}P_3$
42. ${}_6P_6$
43. ${}_6C_6$
44. ${}_8P_4$
45. ${}_8C_4$

46. **CARDS** Four cards are randomly drawn from a standard deck of 52 cards. Find each probability.
  a. P(1 ace and 3 kings)
  b. P(2 even and 2 face cards)

Find the mean, median, and mode for each set of data. Then find the range, variance, and standard deviation for each population.

47. {1, 1, 1, 2, 2, 3}
48. {0.8, 0.9, 0.4, 0.8, 0.6, 0.8, 0.6}

CHAPTER 1

Functions from a Calculus Perspective

©Image Source/Corbis

Then

In **Algebra 2**, you analyzed functions from a graphical perspective.

Now

In **Chapter 1**, you will:

- Explore symmetries of graphs.
- Determine continuity and average rates of change of functions.
- Use limits to describe end behavior.
- Find inverse functions algebraically and graphically.

Why?

BUSINESS Functions are often used throughout the business world. Some of the uses of functions are to analyze costs, predict sales, calculate profit, forecast future costs and revenue, estimate depreciation, and determine the proper labor force.

PREREAD Create a list of two or three things that you already know about functions. Then make a prediction of what you will learn in Chapter 1.

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