

Chapter 8

Rotational Motion

What You'll Learn

- You will learn how to describe and measure rotational motion.
- You will learn how torque changes rotational velocity.
- You will explore factors that determine the stability of an object.
- You will learn the nature of centrifugal and Coriolis “forces.”

Why It's Important

You encounter many rotating objects in everyday life, such as CDs, wheels, and amusement-park rides.

Spin Rides Amusement-park rides that spin are designed to thrill riders using the physics of rotational motion. The thrill is produced by a “force” that is present only when the ride spins.

Think About This ►

Why do people who ride amusement-park rides that spin in circles, such as this one, experience such strong physical reactions?



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How do different objects rotate as they roll?

Question

Do different objects of similar size and mass roll at the same rate on an incline?

Procedure



1. You will need a meterstick, a piece of foam board, a ball, a solid can, and a hollow can.
2. Position the foam board on a 20° incline.
3. Place the meterstick horizontally across the foam board, near the top of the incline, and hold it.
4. Place the ball, solid can, and hollow can against the meterstick. The solid can and hollow can should be placed sideways.
5. Simultaneously, release the three objects by lifting the meterstick.
6. As each object accelerates down the incline, due to gravity, observe the order in which each object reaches the bottom.
7. Repeat steps 2–5 two more times.

Analysis

List the objects in order from the greatest to the least acceleration.

Critical Thinking Which of the objects' properties may have contributed to their behavior? List the properties that were similar and those that were different for each object.



8.1 Describing Rotational Motion

You probably have observed a spinning object many times. How would you measure such an object's rotation? Find a circular object, such as a CD. Mark one point on the edge of the CD so that you can keep track of its position. Rotate the CD to the left (counterclockwise), and as you do so, watch the location of the mark. When the mark returns to its original position, the CD has made one complete revolution. How can you measure a fraction of one revolution? It can be measured in several different ways. A grad is $\frac{1}{400}$ of a revolution, whereas a degree is $\frac{1}{360}$ of a revolution. In mathematics and physics, yet another form of measurement is used to describe fractions of revolutions. In one revolution, a point on the edge travels a distance equal to 2π times the radius of the object. For this reason, the **radian** is defined as $\frac{1}{2}\pi$ of a revolution. In other words, one complete revolution is equal to 2π radians. A radian is abbreviated "rad."

► Objectives

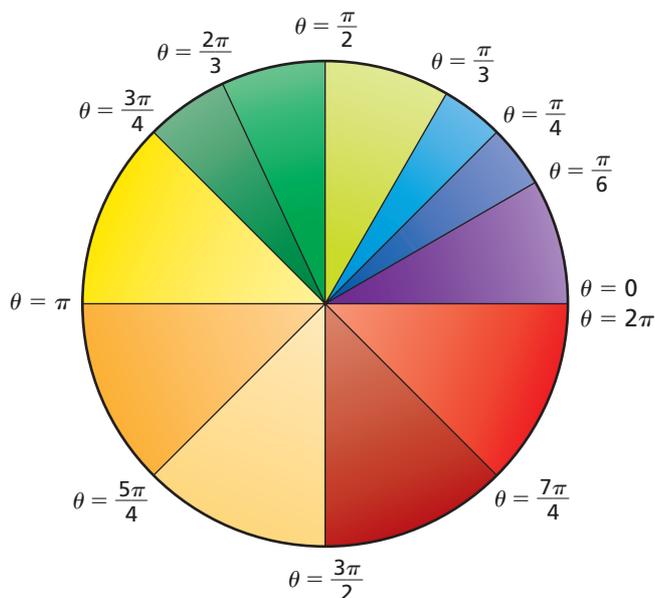
- **Describe** angular displacement.
- **Calculate** angular velocity.
- **Calculate** angular acceleration.
- **Solve** problems involving rotational motion.

► Vocabulary

radian
angular displacement
angular velocity
angular acceleration



■ **Figure 8-1** The pie chart shows the radian measure of the most common angles, measured in the counterclockwise direction. Each angle is measured from $\theta = 0$.



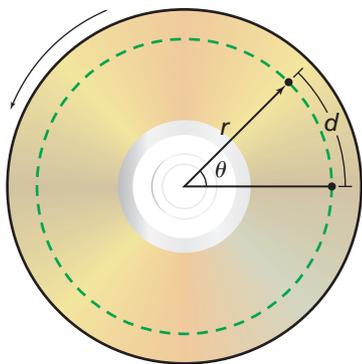
Angular Displacement

The Greek letter theta, θ , is used to represent the angle of revolution. **Figure 8-1** shows the angles in radians for several common fractions of a revolution. Note that counterclockwise rotation is designated as positive, while clockwise is negative. As an object rotates, the change in the angle is called **angular displacement**.

As you know, Earth makes one complete revolution, or 2π rad, in 24 h. In 12 h, its rotation is through π rad. Through what angle does Earth rotate in 6 h? Because 6 h is one-fourth of a day, Earth rotates through an angle of $\frac{\pi}{2}$ rad during that period. Earth's rotation as seen from the north pole is positive. Is it positive or negative when viewed from the south pole?

How far does a point on a rotating object move? You already found that a point on the edge of an object moves 2π times the radius in one revolution. In general, for rotation through an angle, θ , a point at a distance, r , from the center, as shown in **Figure 8-2**, moves a distance given by $d = r\theta$. If r is measured in meters, you might think that multiplying it by θ rad would result in d being measured in m·rad. However, this is not the case. Radians indicate the ratio between d and r . Thus, d is measured in m.

■ **Figure 8-2** The dashed line shows the path of the point on the CD as the CD rotates counterclockwise about its center.



Angular Velocity

How fast does a CD spin? How do you determine its speed of rotation? Recall from Chapter 2 that velocity is displacement divided by the time taken to make the displacement. Likewise, the **angular velocity** of an object is angular displacement divided by the time taken to make the displacement. Thus, the angular velocity of an object is given by the following equation, where angular velocity is represented by the Greek letter omega, ω .

$$\text{Angular Velocity of an Object} \quad \omega = \frac{\Delta\theta}{\Delta t}$$

The angular velocity is equal to the angular displacement divided by the time required to make the rotation.



Recall that if the velocity changes over a time interval, the average velocity is not equal to the instantaneous velocity at any given instant. Similarly, the angular velocity calculated in this way is actually the average angular velocity over a time interval, Δt . Instantaneous angular velocity is equal to the slope of a graph of angular position versus time.

Angular velocity is measured in rad/s. For Earth, $\omega_E = (2\pi \text{ rad}) / (24.0 \text{ h})(3600 \text{ s/h}) = 7.27 \times 10^{-5} \text{ rad/s}$. In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is ω , then the linear velocity of a point a distance, r , from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 464 \text{ m/s}$. Earth is an example of a rotating, rigid body. Even though different points on Earth rotate different distances in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun, on the other hand, is not a rigid body. Different parts of the Sun rotate at different rates. Most objects that we will consider in this chapter are rigid bodies.

Astronomy Connection

Angular Acceleration

What if angular velocity is changing? For example, if a car were accelerated from 0.0 m/s to 25 m/s in 15 s, then the angular velocity of the wheels also would change from 0.0 rad/s to 78 rad/s in the same 15 s. The wheels would undergo **angular acceleration**, which is defined as the change in angular velocity divided by the time required to make the change. Angular acceleration, α , is represented by the following equation.

Angular Acceleration of an Object $\alpha = \frac{\Delta\omega}{\Delta t}$

Angular acceleration is equal to the change in angular velocity divided by the time required to make that change.

Angular acceleration is measured in rad/s^2 . If the change in angular velocity is positive, then the angular acceleration also is positive. Angular acceleration defined in this way is also the average angular acceleration over the time interval Δt . One way to find the instantaneous angular acceleration is to find the slope of a graph of angular velocity as a function of time. The linear acceleration of a point at a distance, r , from the axis of an object with angular acceleration, α , is given by $a = r\alpha$. **Table 8-1** is a summary of linear and angular relationships.

Table 8-1			
Linear and Angular Measures			
Quantity	Linear	Angular	Relationship
Displacement	d (m)	θ (rad)	$d = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s ²)	α (rad/s ²)	$a = r\alpha$



- What is the angular displacement of each of the following hands of a clock in 1 h? State your answer in three significant digits.
 - the second hand
 - the minute hand
 - the hour hand
- If a truck has a linear acceleration of 1.85 m/s^2 and the wheels have an angular acceleration of 5.23 rad/s^2 , what is the diameter of the truck's wheels?
- The truck in the previous problem is towing a trailer with wheels that have a diameter of 48 cm.
 - How does the linear acceleration of the trailer compare with that of the truck?
 - How do the angular accelerations of the wheels of the trailer and the wheels of the truck compare?
- You want to replace the tires on your car with tires that have a larger diameter. After you change the tires, for trips at the same speed and over the same distance, how will the angular velocity and number of revolutions change?

Angular frequency A rotating object can make many revolutions in a given amount of time. For instance, a spinning wheel can go through several complete revolutions in 1 min. Thus, the number of complete revolutions made by the object in 1 s is called angular frequency. Angular frequency is $f = \omega/2\pi$. In the next section, you will explore the factors that cause the angular frequency to change.

8.1 Section Review

- Angular Displacement** A movie lasts 2 h. During that time, what is the angular displacement of each of the following?
 - the hour hand
 - the minute hand
- Angular Velocity** The Moon rotates once on its axis in 27.3 days. Its radius is $1.74 \times 10^6 \text{ m}$.
 - What is the period of the Moon's rotation in seconds?
 - What is the frequency of the Moon's rotation in rad/s?
 - What is the linear speed of a rock on the Moon's equator due only to the Moon's rotation?
 - Compare this speed with the speed of a person on Earth's equator due to Earth's rotation.
- Angular Displacement** The ball in a computer mouse is 2.0 cm in diameter. If you move the mouse 12 cm, what is the angular displacement of the ball?
- Angular Displacement** Do all parts of the minute hand on a watch have the same angular displacement? Do they move the same linear distance? Explain.
- Angular Acceleration** In the spin cycle of a clothes washer, the drum turns at 635 rev/min. If the lid of the washer is opened, the motor is turned off. If the drum requires 8.0 s to slow to a stop, what is the angular acceleration of the drum?
- Critical Thinking** A CD-ROM has a spiral track that starts 2.7 cm from the center of the disk and ends 5.5 cm from the center. The disk drive must turn the disk so that the linear velocity of the track is a constant 1.4 m/s. Find the following.
 - the angular velocity of the disk (in rad/s and rev/min) for the start of the track
 - the disk's angular velocity at the end of the track
 - the disk's angular acceleration if the disk is played for 76 min



8.2 Rotational Dynamics

How do you start the rotation of an object? That is, how do you change its angular velocity? Suppose you have a soup can that you want to spin. If you wrap a string around it and pull hard, you could make the can spin rapidly. Later in this chapter, you will learn why gravity, the force of Earth's mass on the can, acts on the center of the can. The force of the string, on the other hand, is exerted at the outer edge of the can, and at right angles to the line from the center of the can, to the point where the string leaves the can's surface.

You have learned that a force changes the velocity of a point object. In the case of a soup can, a force that is exerted in a very specific way changes the angular velocity of an extended object, which is an object that has a definite shape and size. Consider how you open a door: you exert a force. How can you exert the force to open the door most easily? To get the most effect from the least force, you exert the force as far from the axis of rotation as possible, as shown in **Figure 8-3**. In this case, the axis of rotation is an imaginary vertical line through the hinges. The doorknob is near the outer edge of the door. You exert the force on the doorknob at right angles to the door, away from the hinges. Thus, the magnitude of the force, the distance from the axis to the point where the force is exerted, and the direction of the force determine the change in angular velocity.

Lever arm For a given applied force, the change in angular velocity depends on the **lever arm**, which is the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, as it was with the soup can, then the lever arm is the distance from the axis, r . For the door, it is the distance from the hinges to the point where you exert the force, as illustrated in **Figure 8-4a**, on the next page. If the force is not perpendicular, the perpendicular component of the force must be found.

The force exerted by the string around the can is perpendicular to the radius. If a force is not exerted perpendicular to the radius, however, the lever arm is reduced. To find the lever arm, extend the line of the force until it forms a right angle with a line from the center of rotation. The distance between the intersection and the axis is the lever arm. Thus, using trigonometry, the lever arm, L , can be calculated by the equation $L = r \sin \theta$, as shown in **Figure 8-4b**. In this equation, r is the distance from the axis of rotation to the point where the force is exerted, and θ is the angle between the force and the radius from the axis of rotation to the point where the force is applied.

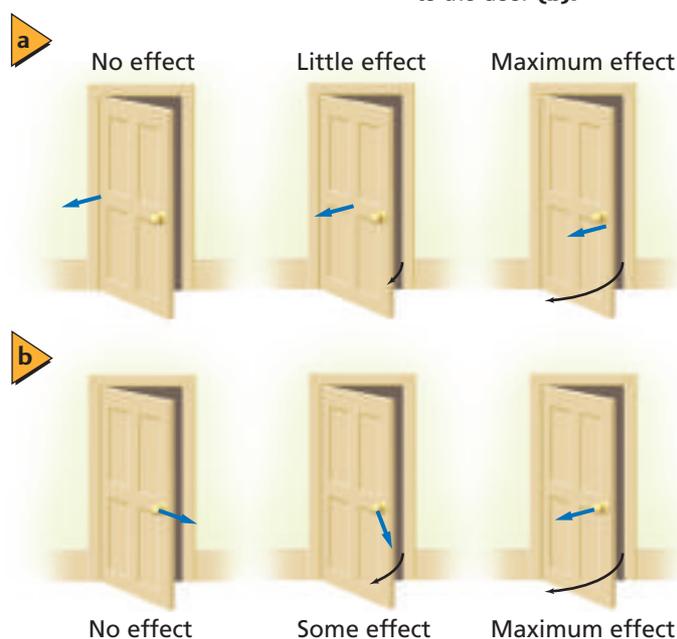
► Objectives

- **Describe** torque and the factors that determine it.
- **Calculate** net torque.
- **Calculate** the moment of inertia.

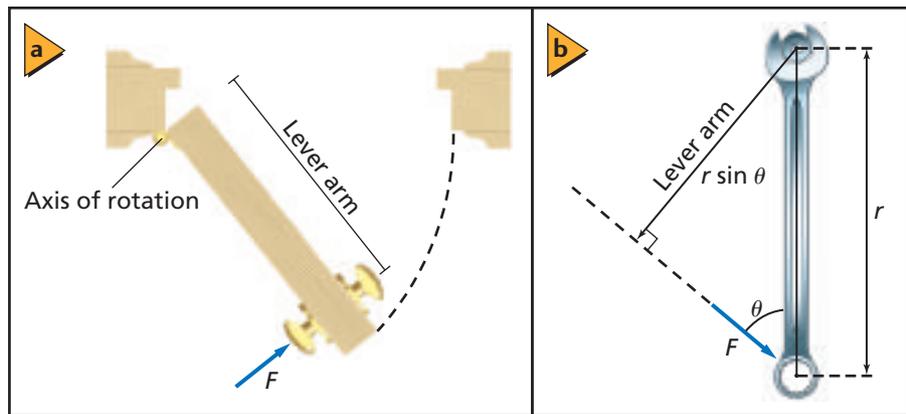
► Vocabulary

lever arm
torque
moment of inertia
Newton's second law for rotational motion

■ **Figure 8-3** When opening a door that is free to rotate about its hinges, the greatest torque is produced when the force is applied farthest from the hinges **(a)**, at an angle perpendicular to the door **(b)**.



■ **Figure 8-4** The lever arm is along the width of the door, from the hinge to the point where the force is exerted **(a)**. The lever arm is equal to $r \sin \theta$, when the angle, θ , between the force and the radius of rotation is not equal to 90° **(b)**.



Torque is a measure of how effectively a force causes rotation. The magnitude of torque is the product of the force and the lever arm. Because force is measured in newtons, and distance is measured in meters, torque is measured in newton-meters ($\text{N}\cdot\text{m}$). Torque is represented by the Greek letter tau, τ . The equation for torque is shown below.

$$\text{Torque } \tau = Fr \sin \theta$$

Torque is equal to the force times the lever arm.

▶ EXAMPLE Problem 1

Lever Arm A bolt on a car engine needs to be tightened with a torque of $35 \text{ N}\cdot\text{m}$. You use a 25-cm -long wrench and pull on the end of the wrench at an angle of 60.0° from the perpendicular. How long is the lever arm, and how much force do you have to exert?

1 Analyze and Sketch the Problem

- Sketch the situation. Find the lever arm by extending the force vector backwards until a line that is perpendicular to it intersects the axis of rotation.

Known:

$$r = 0.25 \text{ m} \quad \tau = 35 \text{ N}\cdot\text{m}$$

$$\theta = 60.0^\circ$$

Unknown:

$$L = ?$$

$$F = ?$$

2 Solve for the Unknown

Solve for the length of the lever arm.

$$L = r \sin \theta$$

$$= (0.25 \text{ m})(\sin 60.0^\circ) \quad \text{Substitute } r = 0.25 \text{ m, } \theta = 60.0^\circ$$

$$= 0.22 \text{ m}$$

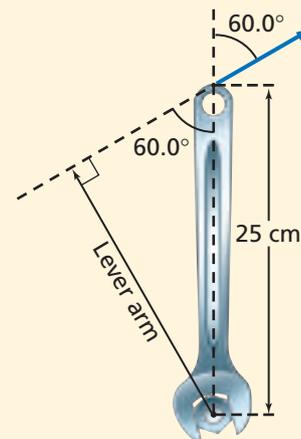
Solve for the force.

$$\tau = Fr \sin \theta$$

$$F = \frac{\tau}{r \sin \theta}$$

$$= \frac{35 \text{ N}\cdot\text{m}}{(0.25 \text{ m})(\sin 60.0^\circ)} \quad \text{Substitute } \tau = 35 \text{ N}\cdot\text{m, } r = 0.25 \text{ m, } \theta = 60.0^\circ$$

$$= 1.6 \times 10^2 \text{ N}$$



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3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Does the sign make sense?** Only the magnitude of the force needed to rotate the wrench clockwise is calculated.

11. Consider the wrench in Example Problem 1. What force is needed if it is applied to the wrench at a point perpendicular to the wrench?
12. If a torque of 55.0 N·m is required and the largest force that can be exerted by you is 135 N, what is the length of the lever arm that must be used?
13. You have a 0.234-m-long wrench. A job requires a torque of 32.4 N·m, and you can exert a force of 232 N. What is the smallest angle, with respect to the vertical, at which the force can be exerted?
14. You stand on the pedal of a bicycle. If you have a mass of 65 kg, the pedal makes an angle of 35° above the horizontal, and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?
15. If the pedal in problem 14 is horizontal, how much torque would you exert? How much torque would you exert when the pedal is vertical?

Finding Net Torque

Try the following experiment. Get two pencils, some coins, and some transparent tape. Tape two identical coins to the ends of the pencil and balance it on the second pencil, as shown in **Figure 8-5**. Each coin exerts a torque that is equal to its weight, F_g , times the distance, r , from the balance point to the center of the coin, as follows:

$$\tau = F_g r$$

But the torques are equal and opposite in direction. Thus, the net torque is zero:

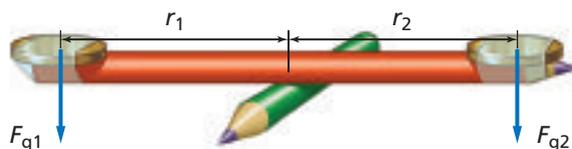
$$\tau_1 - \tau_2 = 0$$

or

$$F_{g1} r_1 - F_{g2} r_2 = 0$$

How can you make the pencil rotate? You could add a second coin on top of one of the two coins, thereby making the two forces different. You also could slide the balance point toward one end or the other of the pencil, thereby making the two distances different.

■ **Figure 8-5** The torque exerted by the first coin, $F_{g1}r_1$, is equal and opposite in direction to the torque exerted by the second coin, $F_{g2}r_2$, when the pencil is balanced.



▶ EXAMPLE Problem 2

Balancing Torques Kariann (56 kg) and Aysha (43 kg) want to balance on a 1.75-m-long seesaw. Where should they place the pivot point?

1 Analyze and Sketch the Problem

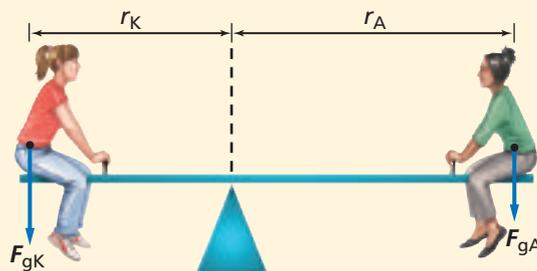
- Sketch the situation.
- Draw and label the vectors.

Known:

$$\begin{aligned} m_K &= 56 \text{ kg} \\ m_A &= 43 \text{ kg} \\ r_K + r_A &= 1.75 \text{ m} \end{aligned}$$

Unknown:

$$\begin{aligned} r_K &= ? \\ r_A &= ? \end{aligned}$$



2 Solve for the Unknown

Find the two forces.

Kariann:

$$\begin{aligned} F_{gK} &= m_K g \\ &= (56 \text{ kg})(9.80 \text{ m/s}^2) && \text{Substitute } m_K = 56 \text{ kg, } g = 9.80 \text{ m/s}^2 \\ &= 5.5 \times 10^2 \text{ N} \end{aligned}$$

Aysha:

$$\begin{aligned} F_{gA} &= m_A g \\ &= (43 \text{ kg})(9.80 \text{ m/s}^2) && \text{Substitute } m_A = 43 \text{ kg, } g = 9.80 \text{ m/s}^2 \\ &= 4.2 \times 10^2 \text{ N} \end{aligned}$$

Define Kariann's distance in terms of the length of the seesaw and Aysha's distance.

$$r_K = 1.75 \text{ m} - r_A$$

When there is no rotation, the sum of the torques is zero.

$$\begin{aligned} F_{gK} r_K &= F_{gA} r_A \\ F_{gK} r_K - F_{gA} r_A &= 0.0 \text{ N}\cdot\text{m} \\ F_{gK}(1.75 \text{ m} - r_A) - F_{gA} r_A &= 0.0 \text{ N}\cdot\text{m} && \text{Substitute } r_K = 1.75 \text{ m} - r_A \end{aligned}$$

Solve for r_A .

$$\begin{aligned} F_{gK}(1.75 \text{ m}) - F_{gK} r_A - F_{gA} r_A &= 0.0 \text{ N}\cdot\text{m} \\ F_{gK} r_A + F_{gA} r_A &= F_{gK}(1.75 \text{ m}) \\ (F_{gK} + F_{gA}) r_A &= F_{gK}(1.75 \text{ m}) \\ r_A &= \frac{F_{gK}(1.75 \text{ m})}{(F_{gK} + F_{gA})} \\ &= \frac{(5.5 \times 10^2 \text{ N})(1.75 \text{ m})}{(5.5 \times 10^2 \text{ N} + 4.2 \times 10^2 \text{ N})} && \text{Substitute } F_{gK} = 5.5 \times 10^2 \text{ N, } F_{gA} = 4.2 \times 10^2 \text{ N} \\ &= 0.99 \text{ m} \end{aligned}$$

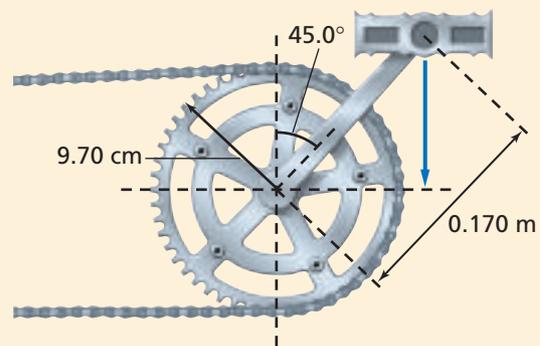
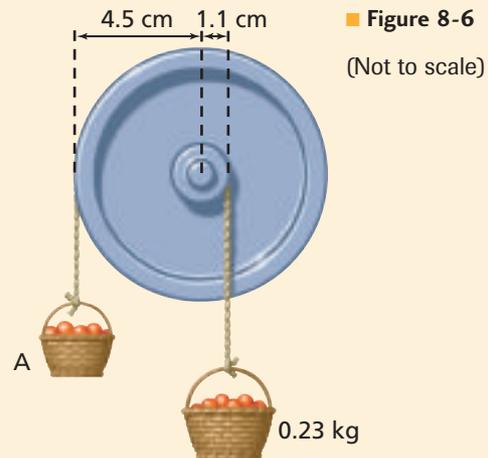
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3 Evaluate the Answer

- **Are the units correct?** Distance is measured in meters.
- **Do the signs make sense?** Distances are positive.
- **Is the magnitude realistic?** Aysha is about 1 m from the center, so Kariann is about 0.75 m away from it. Because Kariann's weight is greater than Aysha's weight, the lever arm on Kariann's side should be shorter. Aysha is farther from the pivot, as expected.

16. Ashok, whose mass is 43 kg, sits 1.8 m from the center of a seesaw. Steve, whose mass is 52 kg, wants to balance Ashok. How far from the center of the seesaw should Steve sit?
17. A bicycle-chain wheel has a radius of 7.70 cm. If the chain exerts a 35.0-N force on the wheel in the clockwise direction, what torque is needed to keep the wheel from turning?
18. Two baskets of fruit hang from strings going around pulleys of different diameters, as shown in **Figure 8-6**. What is the mass of basket A?
19. Suppose the radius of the larger pulley in problem 18 was increased to 6.0 cm. What is the mass of basket A now?
20. A bicyclist, of mass 65.0 kg, stands on the pedal of a bicycle. The crank, which is 0.170 m long, makes a 45.0° angle with the vertical, as shown in **Figure 8-7**. The crank is attached to the chain wheel, which has a radius of 9.70 cm. What force must the chain exert to keep the wheel from turning?



The Moment of Inertia

If you exert a force on a point mass, its acceleration will be inversely proportional to its mass. How does an extended object rotate when a torque is exerted on it? To observe firsthand, recover the pencil, the coins, and the transparent tape that you used earlier in this chapter. First, tape the coins at the ends of the pencil. Hold the pencil between your thumb and forefinger, and wiggle it back and forth. Take note of the forces that your thumb and forefinger exert. These forces create torques that change the angular velocity of the pencil and coins.

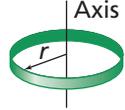
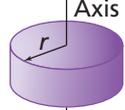
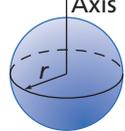
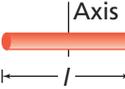
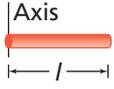
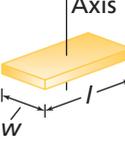
Now move the coins so that they are only 1 or 2 cm apart. Wiggle the pencil as before. Did the amount of torque and force need to be changed? The torque that was required was much less this time. Thus, the amount of mass is not the only factor that determines how much torque is needed to change angular velocity; the location of that mass also is relevant.

The resistance to rotation is called the **moment of inertia**, which is represented by the symbol I and has units of mass times the square of the distance. For a point object located at a distance, r , from the axis of rotation, the moment of inertia is given by the following equation.

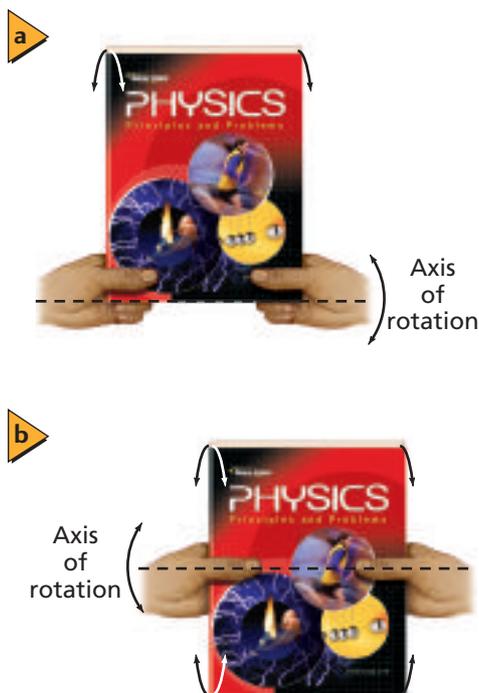
Moment of Inertia of a Point Mass $I = mr^2$

The moment of inertia of a point mass is equal to the mass of the object times the square of the object's distance from the axis of rotation.



Table 8-2			
Moments of Inertia for Various Objects			
Object	Location of Axis	Diagram	Moment of Inertia
Thin hoop of radius r	Through central diameter		mr^2
Solid, uniform cylinder of radius r	Through center		$\frac{1}{2}mr^2$
Uniform sphere of radius r	Through center		$\frac{2}{5}mr^2$
Long, uniform rod of length l	Through center		$\frac{1}{12}ml^2$
Long, uniform rod of length l	Through end		$\frac{1}{3}ml^2$
Thin, rectangular plate of length l and width w	Through center		$\frac{1}{12}m(l^2 + w^2)$

■ **Figure 8-8** The moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book in **(a)** is larger than the moment of inertia of the book in **(b)** because the average distance of the book's mass from the rotational axis is larger.



As you have seen, the moment of inertia for complex objects, such as the pencil and coins, depends on how far the coins are from the axis of rotation. A bicycle wheel, for example, has almost all of its mass in the rim and tire. Its moment of inertia is almost exactly equal to mr^2 , where r is the radius of the wheel. For most objects, however, the mass is distributed continuously and so the moment of inertia is less than mr^2 . For example, as shown in **Table 8-2**, for a solid cylinder of radius r , $I = \frac{1}{2}mr^2$, while for a solid sphere, $I = \frac{2}{5}mr^2$.

The moment of inertia also depends on the location of the rotational axis, as illustrated in **Figure 8-8**. To observe this firsthand, hold a book in the upright position, by placing your hands at the bottom of the book. Feel the torque needed to rock the book towards you, and then away from you. Now put your hands in the middle of the book and feel the torque needed to rock the book toward you and then away from you. Note that much less torque is needed when your hands are placed in the middle of the book because the average distance of the book's mass from the rotational axis is much less in this case.

▶ EXAMPLE Problem 3

Moment of Inertia A simplified model of a twirling baton is a thin rod with two round objects at each end. The length of the baton is 0.65 m, and the mass of each object is 0.30 kg. Find the moment of inertia of the baton if it is rotated about the midpoint between the round objects. What is the moment of inertia of the baton when it is rotated around one end? Which is greater? Neglect the mass of the rod.

1 Analyze and Sketch the Problem

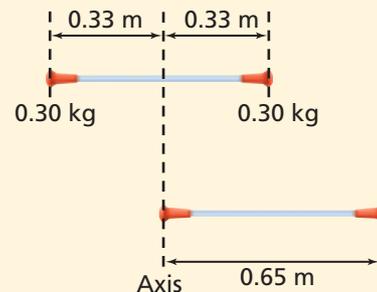
- Sketch the situation. Show the baton with the two different axes of rotation and the distances from the axes of rotation to the masses.

Known: **Unknown:**

$$m = 0.30 \text{ kg}$$

$$I = ?$$

$$l = 0.65 \text{ m}$$



2 Solve for the Unknown

Calculate the moment of inertia of each mass separately.

Rotating about the center of the rod:

$$\begin{aligned} r &= \frac{1}{2}l \\ &= \frac{1}{2}(0.65 \text{ m}) \\ &= 0.33 \text{ m} \end{aligned}$$

Substitute $l = 0.65 \text{ m}$

$$\begin{aligned} I_{\text{single mass}} &= mr^2 \\ &= (0.30 \text{ kg})(0.33 \text{ m})^2 \\ &= 0.033 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $m = 0.30 \text{ kg}$, $r = 0.33 \text{ m}$

Find the moment of inertia of the baton.

$$\begin{aligned} I &= 2I_{\text{single mass}} \\ &= 2(0.033 \text{ kg}\cdot\text{m}^2) \\ &= 0.066 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $I_{\text{single mass}} = 0.033 \text{ kg}\cdot\text{m}^2$

Rotating about one end of the rod:

$$\begin{aligned} I_{\text{single mass}} &= mr^2 \\ &= (0.30 \text{ kg})(0.65 \text{ m})^2 \\ &= 0.13 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $m = 0.30 \text{ kg}$, $r = 0.65 \text{ m}$

Find the moment of inertia of the baton.

$$\begin{aligned} I &= I_{\text{single mass}} \\ &= 0.13 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

The moment of inertia is greater when the baton is swung around one end.

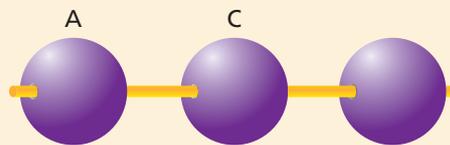
3 Evaluate the Answer

- **Are the units correct?** Moment of inertia is measured in $\text{kg}\cdot\text{m}^2$.
- **Is the magnitude realistic?** Masses and distances are small, and so are the moments of inertia. Doubling the distance increases the moment of inertia by a factor of 4. Thus, doubling the distance overcomes having only one mass contributing.

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21. Two children of equal masses sit 0.3 m from the center of a seesaw. Assuming that their masses are much greater than that of the seesaw, by how much is the moment of inertia increased when they sit 0.6 m from the center?
22. Suppose there are two balls with equal diameters and masses. One is solid, and the other is hollow, with all its mass distributed at its surface. Are the moments of inertia of the balls equal? If not, which is greater?
23. **Figure 8-9** shows three massive spheres on a rod of very small mass. Consider the moment of inertia of the system, first when it is rotated about sphere A, and then when it is rotated about sphere C. Are the moments of inertia the same or different? Explain. If the moments of inertia are different, in which case is the moment of inertia greater?
24. Each sphere in the previous problem has a mass of 0.10 kg. The distance between spheres A and C is 0.20 m. Find the moment of inertia in the following instances: rotation about sphere A, rotation about sphere C.



■ Figure 8-9

Newton's Second Law for Rotational Motion

Newton's second law for linear motion is expressed as $a = F_{\text{net}}/m$. If you rewrite this equation to represent rotational motion, acceleration is replaced by angular acceleration, α , force is replaced by net torque, τ_{net} , and mass is replaced by moment of inertia, I . Thus, **Newton's second law for rotational motion** states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia. This law is expressed by the following equation.

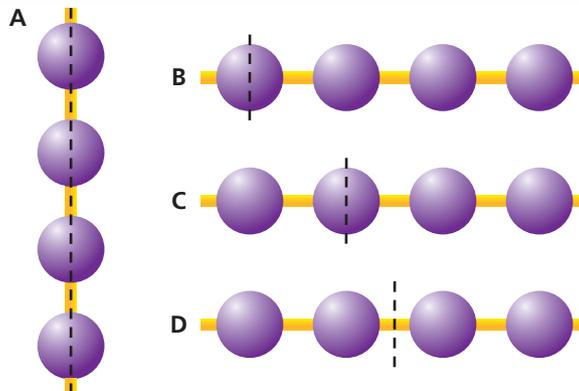
Newton's Second Law for Rotational Motion $\alpha = \frac{\tau_{\text{net}}}{I}$

The angular acceleration of an object is equal to the net torque on the object, divided by the moment of inertia.

Recall the coins taped on the pencil. To change the direction of rotation of the pencil—to give it angular acceleration—you had to apply torque to the pencil. The greater the moment of inertia, the more torque needed to produce the same angular acceleration.

● CHALLENGE PROBLEM

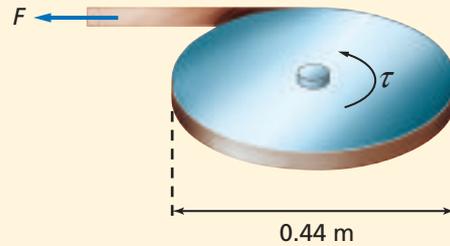
Rank the objects shown in the diagram according to their moments of inertia about the indicated axes. All spheres have equal masses and all separations are the same.



▶ EXAMPLE Problem 4

Torque A solid steel wheel has a mass of 15 kg and a diameter of 0.44 m. It starts at rest. You want to make it rotate at 8.0 rev/s in 15 s.

- What torque must be applied to the wheel?
- If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?



1 Analyze and Sketch the Problem

- Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.

Known:

$$m = 15 \text{ kg}$$

$$r = \frac{1}{2}(0.44 \text{ m}) = 0.22 \text{ m}$$

$$\omega_i = 0.0 \text{ rad/s}$$

$$\omega_f = 2\pi(8.0 \text{ rev/s})$$

$$t = 15 \text{ s}$$

Unknown:

$$\alpha = ?$$

$$I = ?$$

$$\tau = ?$$

$$F = ?$$

2 Solve for the Unknown

- Solve for angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{2\pi(8.0 \text{ rev/s}) - (0.0 \text{ rad/s})}{15 \text{ s}}$$

$$= 3.4 \text{ rad/s}^2$$

Substitute $\omega_f = 2\pi(8.0 \text{ rev/s})$, $\omega_i = 0.0 \text{ rad/s}$

Solve for the moment of inertia.

$$I = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(15 \text{ kg})(0.22 \text{ m})^2$$

$$= 0.36 \text{ kg}\cdot\text{m}^2$$

Substitute $m = 15 \text{ kg}$, $r = 0.22 \text{ m}$

Solve for torque.

$$\tau = I\alpha$$

$$= (0.36 \text{ kg}\cdot\text{m}^2)(3.4 \text{ rad/s}^2)$$

$$= 1.2 \text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$= 1.2 \text{ N}\cdot\text{m}$$

Substitute $I = 0.36 \text{ kg}\cdot\text{m}^2$, $\alpha = 3.4 \text{ rad/s}^2$

- Solve for force.

$$\tau = Fr$$

$$F = \frac{\tau}{r}$$

$$= \frac{1.2 \text{ N}\cdot\text{m}}{0.22 \text{ m}}$$

$$= 5.5 \text{ N}$$

Substitute $\tau = 1.2 \text{ N}\cdot\text{m}$, $r = 0.22 \text{ m}$

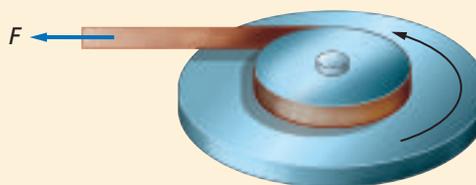
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3 Evaluate the Answer

- Are the units correct?** Torque is measured in N·m and force is measured in N.
- Is the magnitude realistic?** Despite its large mass, the small size of the wheel makes it relatively easy to spin.

25. Consider the wheel in Example Problem 4. If the force on the strap were twice as great, what would be the speed of rotation of the wheel after 15 s?
26. A solid wheel accelerates at 3.25 rad/s^2 when a force of 4.5 N exerts a torque on it. If the wheel is replaced by a wheel with all of its mass on the rim, the moment of inertia is given by $I = mr^2$. If the same angular velocity were desired, what force would have to be exerted on the strap?
27. A bicycle wheel can be accelerated either by pulling on the chain that is on the gear or by pulling on a string wrapped around the tire. The wheel's radius is 0.38 m, while the radius of the gear is 0.14 m. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?
28. The bicycle wheel in problem 27 is used with a smaller gear whose radius is 0.11 m. The wheel can be accelerated either by pulling on the chain that is on the gear or by pulling string that is wrapped around the tire. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?
29. A disk with a moment of inertia of $0.26 \text{ kg}\cdot\text{m}^2$ is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in **Figure 8-10**. Find the force needed to give this system an angular acceleration of 2.57 rad/s^2 .



■ Figure 8-10

In summary, changes in the amount of torque applied to an object, or changes in the moment of inertia, affect the rate of rotation. In this section, you learned how Newton's second law of motion applies to rotational motion. In the next section, you will learn how to keep objects from rotating.

8.2 Section Review

30. **Torque** Vijesh enters a revolving door that is not moving. Explain where and how Vijesh should push to produce a torque with the least amount of force.
31. **Lever Arm** You try to open a door, but you are unable to push at a right angle to the door. So, you push the door at an angle of 55° from the perpendicular. How much harder would you have to push to open the door just as fast as if you were to push it at 90° ?
32. **Net Torque** Two people are pulling on ropes wrapped around the edge of a large wheel. The wheel has a mass of 12 kg and a diameter of 2.4 m. One person pulls in a clockwise direction with a 43-N force, while the other pulls in a counterclockwise direction with a 67-N force. What is the net torque on the wheel?
33. **Moment of Inertia** Refer to Table 8-2 on page 206 and rank the moments of inertia from least to greatest of the following objects: a sphere, a wheel with almost all of its mass at the rim, and a solid disk. All have equal masses and diameters. Explain the advantage of using the one with the least moment of inertia.
34. **Newton's Second Law for Rotational Motion** A rope is wrapped around a pulley and pulled with a force of 13.0 N. The pulley's radius is 0.150 m. The pulley's rotational speed goes from 0.0 to 14.0 rev/min in 4.50 s. What is the moment of inertia of the pulley?
35. **Critical Thinking** A ball on an extremely low-friction, tilted surface, will slide downhill without rotating. If the surface is rough, however, the ball will roll. Explain why, using a free-body diagram.

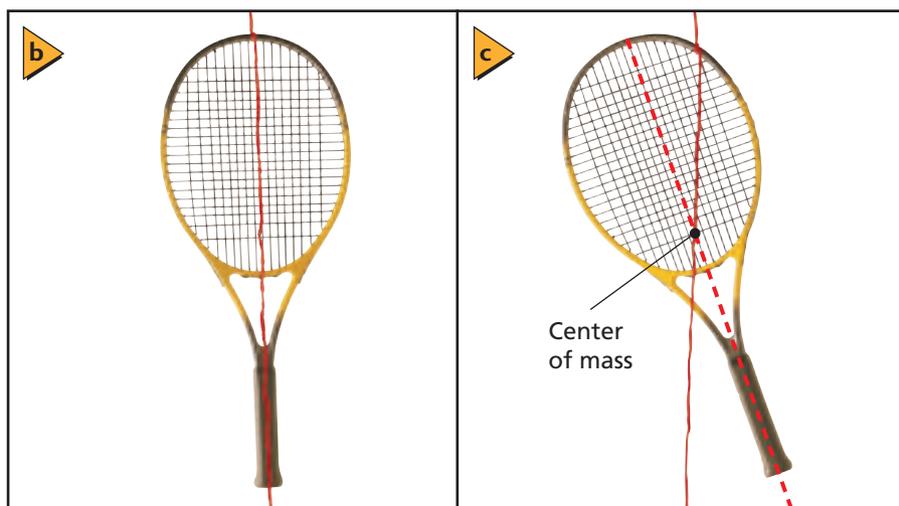
8.3 Equilibrium

Why are some vehicles more likely than others to roll over when involved in an accident? What causes a vehicle to roll over? The answer lies in the design of the vehicle. In this section, you will learn some of the factors that cause an object to tip over.

The Center of Mass

How does an object rotate around its center of mass? A wrench may spin about its handle or end-over-end. Does any single point on the wrench follow a straight path? **Figure 8-11a** shows the path of the wrench. You can see that there is a single point whose path traces a straight line, as if the wrench could be replaced by a point particle at that location. The **center of mass** of an object is the point on the object that moves in the same way that a point particle would move.

Locating the center of mass How can you locate the center of mass of an object? First, suspend the object from any point. When the object stops swinging, the center of mass is along the vertical line drawn from the suspension point as shown in **Figure 8-11b**. Draw the line. Then, suspend the object from another point. Again, the center of mass must be below this point. Draw a second vertical line. The center of mass is at the point where the two lines cross, as shown in **Figure 8-11c**. The wrench, racket, and all other freely rotating objects rotate about an axis that goes through their center of mass. Where is the center of mass of a person located?



▶ Objectives

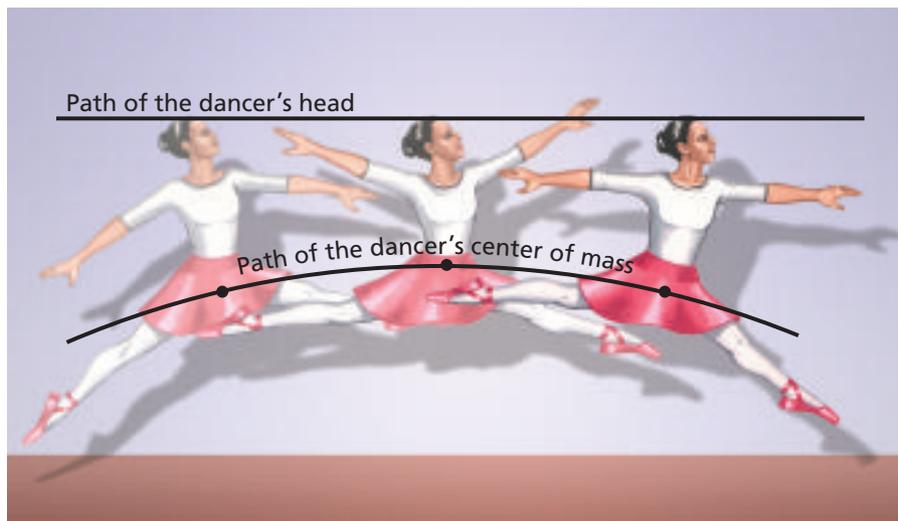
- **Define** center of mass.
- **Explain** how the location of the center of mass affects the stability of an object.
- **Define** the conditions for equilibrium.
- **Describe** how rotating frames of reference give rise to apparent forces.

▶ Vocabulary

center of mass
centrifugal “force”
Coriolis “force”

■ **Figure 8-11** The path of the center of mass of a wrench is a straight line **(a)**. The center of mass of an object such as a tennis racket can be found by first suspending it from any point **(b)**. The point where the strings intersect is the location of the racket’s center of mass **(c)**.

■ **Figure 8-12** The upward motion of the ballet dancer's head is less than the upward motion of the center of mass. Thus, the head and torso move in a nearly horizontal path. This creates an illusion of floating.



APPLYING PHYSICS

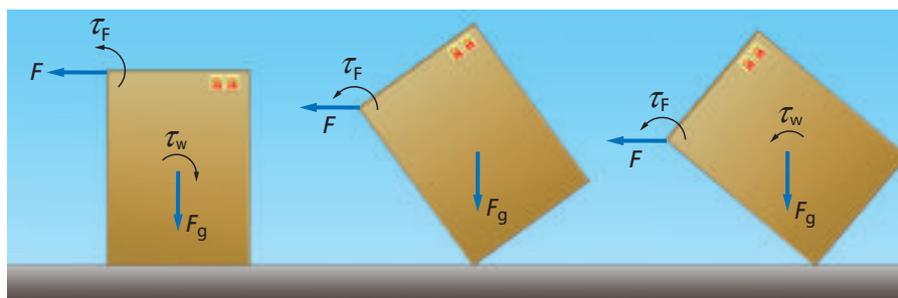
► **The Fosbury-Flop** In high jumping, a technique called the Fosbury-Flop allows a high jumper to clear the bar when it is placed at the highest position. This is possible because the athlete's center of mass passes below the bar as he or she somersaults over the bar, with his or her back toward it. ◀

The center of mass of a human body For a person who is standing with his or her arms hanging straight down, the center of mass is a few centimeters below the navel, midway between the front and back of the person's body. It is slightly higher in young children, because of their relatively larger heads. Because the human body is flexible, however, its center of mass is not fixed. If you raise your hands above your head, your center of mass rises 6 to 10 cm. A ballet dancer, for example, can appear to be floating on air by changing her center of mass in a leap. By raising her arms and legs while in the air, as shown in **Figure 8-12**, the dancer moves her center of mass closer to her head. The path of the center of mass is a parabola, so the dancer's head stays at almost the same height for a surprisingly long time.

Center of Mass and Stability

What factors determine whether a vehicle is stable or prone to roll over in an accident? To understand the problem, think about tipping over a box. A tall, narrow box, standing on end, tips more easily than a low, broad box. Why? To tip a box, as shown in **Figure 8-13**, you must rotate it about a corner. You pull at the top with a force, F , applying a torque, τ_F . The weight of the box, acting on the center of mass, F_g , applies an opposing torque, τ_w . When the center of mass is directly above the point of support, τ_w is zero. The only torque is the one applied by you. As the box rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. At this point, the box tips over rapidly.

■ **Figure 8-13** The bent arrows show the direction of the torque produced by the force exerted to tip over a box.



Spinning Tops 

1. Cut out two cardboard disks of 10-cm and 15-cm diameter.
2. Use a pencil with an eraser that has rounded edges from use. If it is new, rub it on paper to round it.
3. Spin the pencil and try to make it stand on the eraser. Repeat several times and record your observations.
4. Carefully push the pencil through the center of the 10-cm disk.
5. Spin the pencil with the disk and try to make it stand on the eraser.
6. Move the disk to different points on the pencil. Spin and record your observations.
7. Repeat steps 4-6 with the 15-cm disk.

Analyze and Conclude

8. **Sequence** the three trials in order from least to most stable.
9. **Describe** the location of the pencil's center of mass.
10. **Analyze** the placement of the disk and its effect on stability.

Stability An object is said to be stable if an external force is required to tip it. The box in Figure 8-13 is stable as long as the direction of the torque due to its weight, τ_w tends to keep it upright. This occurs as long as the box's center of mass lies above its base. To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer above the base of the box. To rotate the box, you must lift its center of mass. The broader the base, the more stable the object is. For this reason, if you are standing on a bus that is weaving through traffic and you want to avoid falling down, you need to stand with your feet spread apart.

Why do vehicles roll over? **Figure 8-14** shows two vehicles rolling over. Note that the one with the higher center of mass does not have to be tilted very far for its center of mass to be outside its base—its center of mass does not have to be raised as much as the other vehicle's. The lower the location of an object's center of mass, the greater its stability.

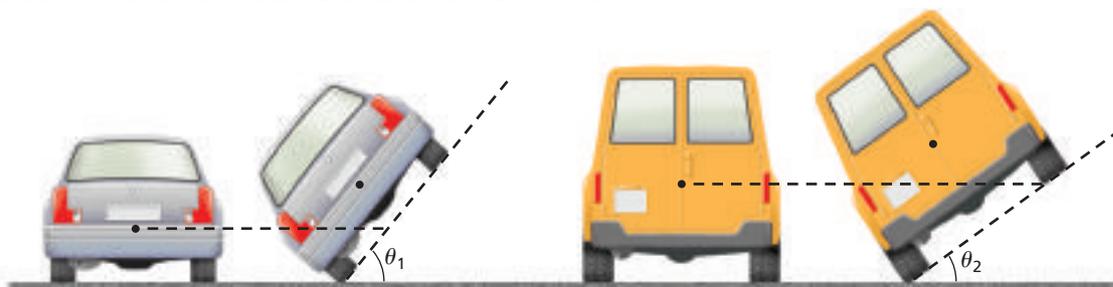
You are stable when you stand flat on your feet. When you stand on tiptoe, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. A small person can use torque, rather than force, to defend himself or herself against a stronger person. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position, where the opponent's center of mass does not lie above his or her feet.

In summary, if the center of mass is outside the base of an object, it is unstable and will roll over without additional torque. If the center of mass is above the base of the object, it is stable. If the base of the object is very narrow and the center of mass is high, then the object is stable, but the slightest force will cause it to tip over.

Conditions for Equilibrium

If your pen is at rest, what is needed to keep it at rest? You could either hold it up or place it on a desk or some other surface. An upward force must be exerted on the pen to balance the downward force of gravity. You must also hold the pen so that it will not rotate. An object is said to be in static equilibrium if both its velocity and angular velocity are zero or constant. Thus, for an object to be in static equilibrium, it must meet two conditions. First, it must be in translational equilibrium; that is, the net force exerted on the object must be zero. Second, it must be in rotational equilibrium; that is, the net torque exerted on the object must be zero.

■ **Figure 8-14** Larger vehicles have a higher center of mass than smaller ones. The higher the center of mass, the smaller the tilt needed to cause the vehicle's center of mass to move outside its base and cause the vehicle to roll over.



▶ EXAMPLE Problem 5

Static Equilibrium A 5.8-kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from one end of the ladder, and sawhorse B is 0.15 m from the other end of the ladder. What force does each sawhorse exert on the ladder?

1 Analyze and Sketch the Problem

- Sketch the situation.
- Choose the axis of rotation at the point where F_A acts on the ladder. Thus, the torque due to F_A is zero.

Known:

$$\begin{aligned} m &= 5.8 \text{ kg} \\ l &= 1.80 \text{ m} \\ l_A &= 0.60 \text{ m} \\ l_B &= 0.15 \text{ m} \end{aligned}$$

Unknown:

$$\begin{aligned} F_A &= ? \\ F_B &= ? \end{aligned}$$

2 Solve for the Unknown

For a ladder that has a constant density, the center of mass is at the center rung.

The net force is the sum of all forces on the ladder.

$$\begin{aligned} F_{\text{net}} &= F_A + F_B + (-F_g) && \text{The ladder is in translational equilibrium, so the net force exerted on it is zero.} \\ 0.0 \text{ N} &= F_A + F_B - F_g \end{aligned}$$

Solve for F_A .

$$F_A = F_g - F_B$$

Find the torques due to F_g and F_B .

$$\begin{aligned} \tau_g &= -r_g F_g && \tau_g \text{ is in the clockwise direction.} \\ \tau_B &= +r_B F_B && \tau_B \text{ is in the counterclockwise direction.} \end{aligned}$$

The net torque is the sum of all torques on the object.

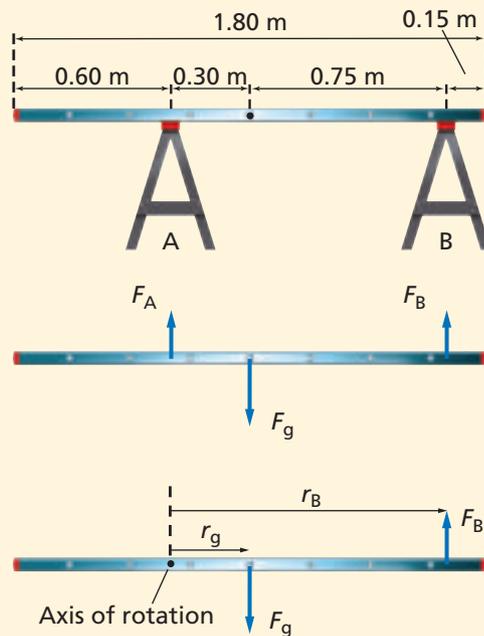
$$\begin{aligned} \tau_{\text{net}} &= \tau_B + \tau_g \\ 0.0 \text{ N}\cdot\text{m} &= \tau_B + \tau_g && \text{The ladder is in rotational equilibrium, so } \tau_{\text{net}} = 0.0 \text{ N}\cdot\text{m.} \\ \tau_B &= \tau_g \\ r_B F_B &= r_g F_g && \text{Substitute } \tau_B = r_B F_B, \tau_g = -r_g F_g \end{aligned}$$

Solve for F_B .

$$\begin{aligned} F_B &= \frac{r_g F_g}{r_B} \\ &= \frac{r_g mg}{r_B} && \text{Substitute } F_g = mg \end{aligned}$$

Using the expression $F_A = F_g - F_B$, substitute in the expressions for F_B and F_g .

$$\begin{aligned} F_A &= F_g - F_B \\ &= F_g - \frac{r_g mg}{r_B} && \text{Substitute } F_B = \frac{r_g mg}{r_B} \\ &= mg - \frac{r_g mg}{r_B} && \text{Substitute } F_g = mg \\ &= mg \left(1 - \frac{r_g}{r_B} \right) \end{aligned}$$



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Solve for r_g .

$$\begin{aligned}r_g &= \frac{1}{2} l_A \\ &= 0.90 \text{ m} - 0.60 \text{ m} \\ &= 0.30 \text{ m}\end{aligned}$$

For a ladder, which has a constant density, the center of mass is at the center rung.

Substitute $\frac{l}{2} = 0.90 \text{ m}$, $l_A = 0.60 \text{ m}$

Solve for r_B .

$$\begin{aligned}r_B &= (0.90 \text{ m} - l_B) + (0.90 \text{ m} - l_A) \\ &= (0.90 \text{ m} - 0.15 \text{ m}) + (0.90 \text{ m} - 0.60 \text{ m}) \\ &= 0.75 \text{ m} + 0.30 \text{ m} \\ &= 1.05 \text{ m}\end{aligned}$$

Substitute $l_B = 0.15 \text{ m}$, $l_A = 0.60 \text{ m}$

Calculate F_B .

$$\begin{aligned}F_B &= \frac{r_g mg}{r_B} \\ &= \frac{(0.30 \text{ m})(5.8 \text{ kg})(9.80 \text{ m/s}^2)}{(1.05 \text{ m})} \\ &= 16 \text{ N}\end{aligned}$$

Substitute $r_g = 0.30 \text{ m}$, $m = 5.8 \text{ kg}$, $g = 9.80 \text{ m/s}^2$, $r_B = 1.05 \text{ m}$

Calculate F_A .

$$\begin{aligned}F_A &= mg \left(1 - \frac{r_g}{r_B}\right) \\ &= \left(1 - \frac{(0.30 \text{ m})}{(1.05 \text{ m})}\right)(5.8 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 41 \text{ N}\end{aligned}$$

Substitute $r_g = 0.30 \text{ m}$, $m = 5.8 \text{ kg}$, $g = 9.80 \text{ m/s}^2$, $r_B = 1.05 \text{ m}$

3 Evaluate the Answer

- **Are the units correct?** Forces are measured in newtons.
- **Do the signs make sense?** Both forces are upward.
- **Is the magnitude realistic?** The forces add up to the weight of the ladder, and the force exerted by the sawhorse closer to the center of mass is greater, which is correct.

PRACTICE Problems

Additional Problems, Appendix B

36. What would be the forces exerted by the two sawhorses if the ladder in Example Problem 5 had a mass of 11.4 kg?

37. A 73-kg ladder, 1.92 m long, rests on two sawhorses, as shown in **Figure 8-15**. Sawhorse A, on the left, is located 0.30 m from the end, and sawhorse B, on the right, is located 0.45 m from the other end. Choose the axis of rotation to be the center of mass of the ladder.

- What are the torques acting on the ladder?
- Write the equation for rotational equilibrium.
- Solve the equation for F_A in terms of F_g .
- How would the forces exerted by the two sawhorses change if A were moved very close to, but not directly under, the center of mass?

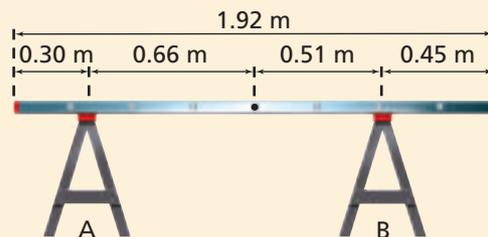


Figure 8-15

38. A 4.5-m-long wooden plank with a 24-kg mass is supported in two places. One support is directly under the center of the board, and the other is at one end. What are the forces exerted by the two supports?

39. A 85-kg diver walks to the end of a diving board. The board, which is 3.5 m long with a mass of 14 kg, is supported at the center of mass of the board and at one end. What are the forces on the two supports?

Rotating Frames of Reference

When you are on a rapidly spinning amusement-park ride, it feels like a strong force is pushing you to the outside. A pebble on the floor of the ride would accelerate outward without a horizontal force being exerted on it in the same direction. The pebble would not move in a straight line. In other words, Newton's laws would not apply. This is because rotating frames of reference are accelerated frames. Newton's laws are valid only in inertial or nonaccelerated frames.

Motion in a rotating reference frame is important to us because Earth rotates. The effects of the rotation of Earth are too small to be noticed in the classroom or lab, but they are significant influences on the motion of the atmosphere and therefore, on climate and weather.

Centrifugal "Force"

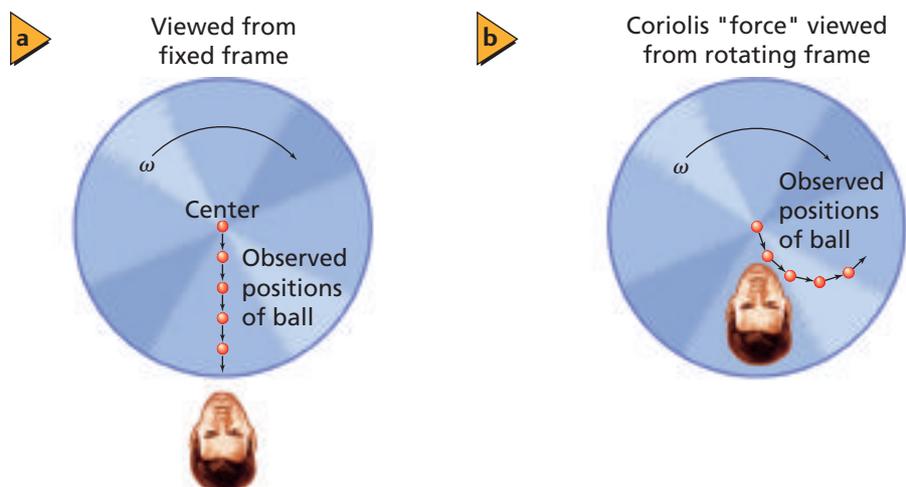
Suppose you fasten one end of a spring to the center of a rotating platform. An object lies on the platform and is attached to the other end of the spring. As the platform rotates, an observer on the platform sees the object stretch the spring. The observer might think that some force toward the outside of the platform is pulling on the object. This apparent force is called **centrifugal "force."** It is not a real force because there is no physical outward push on the object. Still, this "force" seems real, as anyone who has ever been on an amusement-park ride can attest.

As the platform rotates, an observer on the ground sees things differently. This observer sees the object moving in a circle. The object accelerates toward the center because of the force of the spring. As you know, the acceleration is centripetal acceleration and is given by $a_c = v^2/r$. It also can be written in terms of angular velocity, as $a_c = \omega^2/r$. Centripetal acceleration is proportional to the distance from the axis of rotation and depends on the square of the angular velocity. Thus, if you double the rotational frequency, the acceleration increases by a factor of 4.

The Coriolis "Force"

A second effect of rotation is shown in **Figure 8-16**. Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. Consider the horizontal motion of the ball as seen by two observers and ignore the vertical motion of the ball as it falls.

■ **Figure 8-16** The Coriolis "force" exists only in rotating reference frames.



An observer standing outside the disk, as shown in **Figure 8-16a**, sees the ball travel in a straight line at a constant speed toward the edge of the disk. However, the other observer, who is stationed on the disk and rotating with it, as shown in **Figure 8-16b**, sees the ball follow a curved path at a constant speed. A force seems to be acting to deflect the ball. This apparent force is called the **Coriolis “force.”** Like the centrifugal “force,” the Coriolis “force” is not a real force. It seems to exist because we observe a deflection in horizontal motion when we are in a rotating frame of reference.

Coriolis “force” due to Earth Suppose a cannon is fired from a point on the equator toward a target due north of it. If the projectile were fired directly northward, it would also have an eastward velocity component because of the rotation of Earth. This eastward speed is greater at the equator than at any other latitude. Thus, as the projectile moves northward, it also moves eastward faster than points on Earth below it do. The result is that the projectile lands east of the target as shown in **Figure 8-17**. While an observer in space would see Earth’s rotation, an observer on Earth could claim that the projectile missed the target because of the Coriolis “force” on the rocket. Note that for objects moving toward the equator, the direction of the apparent force is westward. A projectile will land west of the target when fired due south.

The direction of winds around high- and low-pressure areas results from the Coriolis “force.” Winds flow from areas of high to low pressure. Because of the Coriolis “force” in the northern hemisphere, winds from the south go to the east of low-pressure areas. Winds from the north, however, end up west of low-pressure areas. Therefore, winds rotate counterclockwise around low-pressure areas in the northern hemisphere. In the southern hemisphere however, winds rotate clockwise around low-pressure areas.

Most amusement-park rides thrill the riders because they are in accelerated reference frames while on the ride. The “forces” felt by roller-coaster riders at the tops and bottoms of hills, and when moving almost vertically downward, are mostly related to linear acceleration. On Ferris wheels, rotors, other circular rides, and on the curves of roller coasters, centrifugal “forces” provide most of the excitement.



■ **Figure 8-17** An observer on Earth sees the Coriolis “force” cause a projectile fired due north to deflect to the right of the intended target.

Meteorology Connection

8.3 Section Review

40. **Center of Mass** Can the center of mass of an object be located in an area where the object has no mass? Explain.
41. **Stability of an Object** Why is a modified vehicle with its body raised high on risers less stable than a similar vehicle with its body at normal height?
42. **Conditions for Equilibrium** Give an example of an object for each of the following conditions.
 - a. rotational equilibrium, but not translational equilibrium
 - b. translational equilibrium, but not rotational equilibrium
43. **Center of Mass** Where is the center of mass of a roll of masking tape?
44. **Locating the Center of Mass** Describe how you would find the center of mass of this textbook.
45. **Rotating Frames of Reference** A penny is placed on a rotating, old-fashioned record turntable. At the highest speed, the penny starts sliding outward. What are the forces acting on the penny?
46. **Critical Thinking** You have learned why the winds around a low-pressure area move in a counterclockwise direction. Would the winds move in the same or opposite direction in the southern hemisphere? Explain.



Translational and Rotational Equilibrium

Alternate CBL instructions can be found on the Web site.
physicspp.com

For maintenance on large buildings, scaffolding can be hung on the outside. In order for the scaffolding to support workers, it must be in translational and rotational equilibrium. If two or more forces act on the scaffolding, each can produce a rotation about either end. Scaffolding with uniform mass distribution acts as though all of the mass is concentrated at its center. In translational equilibrium the object is not accelerating; thus, the upward and downward forces are equal.

In order to achieve rotational equilibrium, the sum of all the clockwise torques must equal the sum of all the counterclockwise torques as measured from a pivot point. That is, the net torque must be zero. In this lab you will model scaffolding hung from two ropes using a meterstick and spring scales, and use numbers to measure the forces on the scaffolding.

QUESTION

What conditions are required for equilibrium when parallel forces act on an object?

Objectives

- **Collect and organize data** about the forces acting on the scaffolding.
- **Describe** clockwise and counterclockwise torque.
- **Compare and contrast** translational and rotational equilibrium.

Safety Precautions



- **Use care to avoid dropping masses.**

Materials

meterstick	two Buret clamps
two 0–5 N spring scales	500-g hooked mass
two ring stands	200-g hooked mass

Procedure

The left spring scale will be considered a pivot point for the purposes of this lab. Therefore, the lever arm will be measured from this point.

1. Place the ring stands 80 cm apart.
2. Attach a Buret clamp to each of the ring stands.
3. Verify that the scales are set to zero before use. If the scales need to be adjusted, ask your teacher for assistance.
4. Hang a spring scale from each Buret clamp attached to a ring stand.
5. Hook the meterstick onto the spring scale in such a manner that the 10-cm mark is supported by one hook and the 90-cm mark is supported by the other hook.
6. Read each spring scale and record the force in Data Table 1.
7. Hang a 500-g mass on the meterstick at the 30-cm mark. This point should be 20-cm from the left scale.
8. Read each spring scale and record the force in Data Table 1.
9. Hang a 200-g mass on the meterstick at the 70-cm mark. This point should be 60 cm from the left scale.
10. Read each spring scale and record the force in Data Table 1.



Data Table 1			
Object Added	Distance From Left Scale (m)	Left Scale Reading (N)	Right Scale Reading (N)
Meterstick	0.4		
500-g mass	0.2		
200-g mass	0.6		

Data Table 2				
Object Added	τ_c	τ_{cc}	Lever Arm (m)	Force (N)
Meterstick				
500-g mass				
200-g mass				
Right scale				

Data Table 3		
Object Added	τ_c (N·m)	τ_{cc} (N·m)
Meterstick		
500-g mass		
200-g mass		
Right scale		
$\Sigma\tau$		

Analyze

- Calculate** Find the mass of the meterstick.
- Calculate** Find the force, or weight, that results from each object and record it in Data Table 2. For the right scale, read the force it exerts and record it in Data Table 2.
- Using the point where the left scale is attached as a pivot point, identify the forces located elsewhere that cause the scaffold to rotate clockwise or counterclockwise. Mark these in Data Table 2 with an x .
- Record the lever arm distance of each force from the pivot point in Data Table 2.
- Use Numbers** Calculate the torque for each object by multiplying the force and lever arm distance. Record these values in Data Table 3.
- Compare and contrast the sum of the clockwise torques, $\Sigma\tau_c$, and the counterclockwise torques, $\Sigma\tau_{cc}$.
- What is the percent difference between $\Sigma\tau_c$ and $\Sigma\tau_{cc}$?

Going Further

Use additional masses at locations of your choice with your teacher's permission and record your data.

Real-World Physics

Research the safety requirements in your area for putting up, using, and dismantling scaffolding.

Conclude and Apply

- Is the system in translational equilibrium? How do you know?
- Draw a free-body diagram of your system, showing all the forces.



To find out more about rotational motion, visit the Web site: physicspp.com

The Stability of Sport-Utility Vehicles

Why are sport-utility vehicles more flippable? Many believe that the large size of the sport-utility vehicle makes it more stable and secure. But, a sport-utility vehicle, as well as other tall vehicles such as vans, is much more likely to roll over than a car.

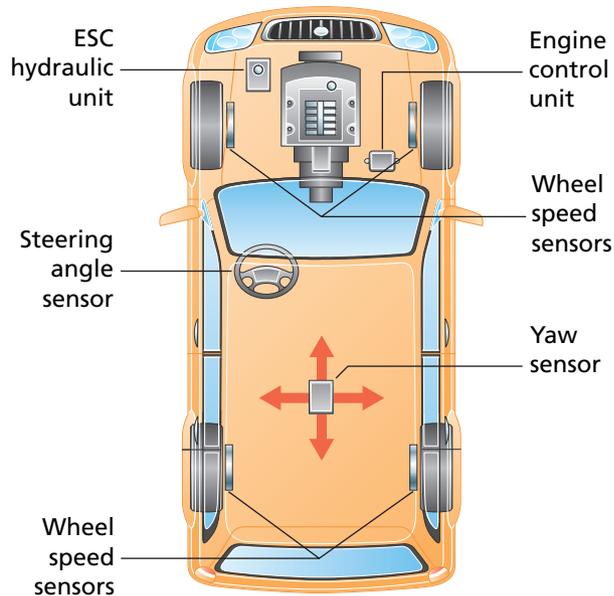
The Problem A sport-utility vehicle has a high center of mass which makes it more likely to topple. Another factor that affects rollover is the static stability factor, which is the ratio of the track width to the center of mass. Track width is defined as half the distance between the two front wheels. The higher the static stability factor, the more likely a vehicle will stay upright.

Many sport-utility vehicles have a center of mass 13 or 15 cm higher than passenger cars. Their track width, however, is about the same as that of passenger cars. Suppose the stability factor for a sport-utility vehicle is 1.06 and 1.43 for a car. Statistics show that in a single-vehicle crash, the sport-utility vehicle has a 37 percent chance of rolling over, while the car has a 10.6 percent chance of rolling over.

However, the static stability factor oversimplifies the issue. Weather and driver behavior are also contributors to rollover crashes. Vehicle factors, such as tires, suspension systems, inertial properties, and advanced handling systems all play a role as well.

It is true that most rollover crashes occur when a vehicle swerves off the road and hits a rut, soft soil, or other surface irregularity. This usually occurs when a driver is not paying proper attention or is speeding. Safe drivers greatly reduce their chances of being involved in a rollover accident by paying attention and driving at the correct speed. Still, weather and driver behavior being equal, the laws of physics indicate that sport-utility vehicles carry an increased risk.

What Is Being Done? Some models are being built with wider track widths or stronger roofs. Optional side-curtain air bags have sensors to keep the bags inflated for up to 6 s, rather than the usual fraction of a second. This will cushion passengers if the vehicle should flip several times.



An ESC system processes information from the sensors and automatically applies the brakes to individual wheels when instability is detected.

A promising new technology called Electronic Stability Control (ESC) can be used to prevent rollover accidents. An ESC system has electronic sensors that detect when a vehicle begins to spin due to oversteering, and also when it begins to slide in a plowlike manner because of understeering. In these instances, an ESC system automatically applies the brakes at one or more wheels, thereby reorienting the vehicle in the right direction.

Safe driving is the key to preventing many automobile accidents. Knowledge of the physics behind rollover accidents and the factors that affect rollover accidents may help make you an informed, safe driver.

Going Further

- 1. Hypothesize** In a multi-vehicle accident, sport-utility vehicles generally fare better than the passenger cars involved in the accident. Why is this so?
- 2. Debate the Issue** ESC is a life-saving technology. Should it be mandatory in all sport-utility vehicles? Why or why not?

8.1 Describing Rotational Motion

Vocabulary

- radian (p. 197)
- angular displacement (p. 198)
- angular velocity (p. 198)
- angular acceleration (p. 199)

Key Concepts

- Angular position and its changes are measured in radians. One complete revolution is 2π rad.
- Angular velocity is given by the following equation.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Angular acceleration is given by the following equation.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- For a rotating, rigid object, the angular displacement, velocity, and acceleration can be related to the linear displacement, velocity, and acceleration for any point on the object.

$$d = r\theta \quad v = r\omega \quad a = r\alpha$$

8.2 Rotational Dynamics

Vocabulary

- lever arm (p. 201)
- torque (p. 202)
- moment of inertia (p. 205)
- Newton's second law for rotational motion (p. 208)

Key Concepts

- When torque is exerted on an object, its angular velocity changes.
- Torque depends on the magnitude of the force, the distance from the axis of rotation at which it is applied, and the angle between the force and the radius from the axis of rotation to the point where the force is applied.

$$\tau = Fr \sin \theta$$

- The moment of inertia of an object depends on the way the object's mass is distributed about the rotational axis. For a point object:

$$I = mr^2$$

- Newton's second law for rotational motion states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia.

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

8.3 Equilibrium

Vocabulary

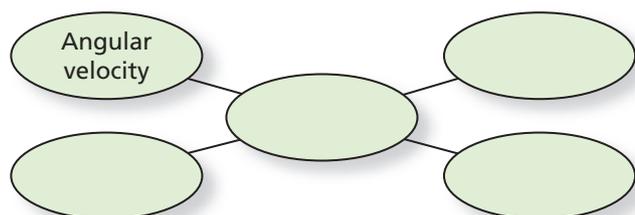
- center of mass (p. 211)
- centrifugal "force" (p. 216)
- Coriolis "force" (p. 217)

Key Concepts

- The center of mass of an object is the point on the object that moves in the same way that a point particle would move.
- An object is stable against rollover if its center of mass is above its base.
- An object is in equilibrium if there are no net forces exerted on it and if there are no net torques acting on it.
- Centrifugal "force" and the Coriolis "force" are two apparent forces that appear when a rotating object is analyzed from a coordinate system that rotates with it.

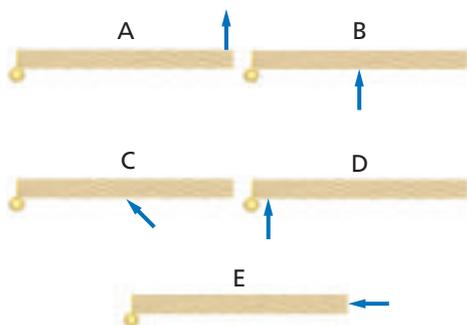
Concept Mapping

47. Complete the following concept map using the following terms: *angular acceleration*, *radius*, *tangential acceleration*, *centripetal acceleration*.



Mastering Concepts

48. A bicycle wheel rotates at a constant 25 rev/min. Is its angular velocity decreasing, increasing, or constant? (8.1)
49. A toy rotates at a constant 5 rev/min. Is its angular acceleration positive, negative, or zero? (8.1)
50. Do all parts of Earth rotate at the same rate? Explain. (8.1)
51. A unicycle wheel rotates at a constant 14 rev/min. Is the total acceleration of a point on the tire inward, outward, tangential, or zero? (8.1)
52. Think about some possible rotations of your textbook. Are the moments of inertia about these three axes the same or different? Explain. (8.2)
53. Torque is important when tightening bolts. Why is force not important? (8.2)
54. Rank the torques on the five doors shown in **Figure 8-18** from least to greatest. Note that the magnitude of all the forces is the same. (8.2)



■ Figure 8-18

55. Explain how you can change an object's angular frequency. (8.2)
56. To balance a car's wheel, it is placed on a vertical shaft and weights are added to make the wheel horizontal. Why is this equivalent to moving the center of mass until it is at the center of the wheel? (8.3)

57. A stunt driver maneuvers a monster truck so that it is traveling on only two wheels. Where is the center of mass of the truck? (8.3)
58. Suppose you stand flat-footed, then you rise and balance on tiptoe. If you stand with your toes touching a wall, you cannot balance on tiptoe. Explain. (8.3)
59. Why does a gymnast appear to be floating on air when she raises her arms above her head in a leap? (8.3)
60. Why is a vehicle with wheels that have a large diameter more likely to roll over than a vehicle with wheels that have a smaller diameter? (8.3)

Applying Concepts

61. Two gears are in contact and rotating. One is larger than the other, as shown in **Figure 8-19**. Compare their angular velocities. Also compare the linear velocities of two teeth that are in contact.



■ Figure 8-19

62. **Videotape** When a videotape is rewound, why does it wind up fastest towards the end?
63. **Spin Cycle** What does a spin cycle of a washing machine do? Explain in terms of the forces on the clothes and water.
64. How can you experimentally find the moment of inertia of an object?
65. **Bicycle Wheels** Three bicycle wheels have masses that are distributed in three different ways: mostly at the rim, uniformly, and mostly at the hub. The wheels all have the same mass. If equal torques are applied to them, which one will have the greatest angular acceleration? Which one will have the least?
66. **Bowling Ball** When a bowling ball leaves a bowler's hand, it does not spin. After it has gone about half the length of the lane, however, it does spin. Explain how its rotation rate increased and why it does not continue to increase.

67. **Flat Tire** Suppose your car has a flat tire. You get out your tools and find a lug wrench to remove the nuts off the bolt studs. You find it impossible to turn the nuts. Your friend suggests ways you might produce enough torque to turn them. What three ways might your friend suggest?
68. **Tightrope Walkers** Tightrope walkers often carry long poles that sag so that the ends are lower than the center as shown in **Figure 8-20**. How does such a pole increase the tightrope walker's stability? *Hint: Consider both center of mass and moment of inertia.*



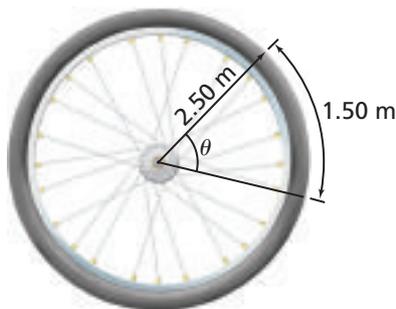
■ Figure 8-20

69. **Merry-Go-Round** While riding a merry-go-round, you toss a key to a friend standing on the ground. For your friend to be able to catch the key, should you toss it a second or two before you reach the spot where your friend is standing or wait until your friend is directly behind you? Explain.
70. Why can you ignore forces that act on the axis of rotation of an object in static equilibrium when determining the net torque?
71. In solving problems about static equilibrium, why is the axis of rotation often placed at a point where one or more forces are acting on the object?

Mastering Problems

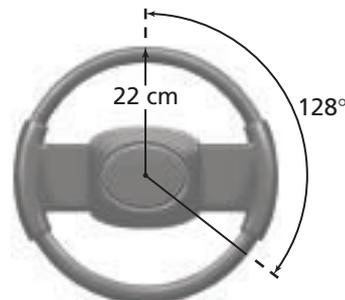
8.1 Describing Rotational Motion

72. A wheel is rotated so that a point on the edge moves through 1.50 m. The radius of the wheel is 2.50 m, as shown in **Figure 8-21**. Through what angle (in radians) is the wheel rotated?



■ Figure 8-21

73. The outer edge of a truck tire that has a radius of 45 cm has a velocity of 23 m/s. What is the angular velocity of the tire in rad/s?
74. A steering wheel is rotated through 128° , as shown in **Figure 8-22**. Its radius is 22 cm. How far would a point on the steering wheel's edge move?



■ Figure 8-22

75. **Propeller** A propeller spins at 1880 rev/min.
- What is its angular velocity in rad/s?
 - What is the angular displacement of the propeller in 2.50 s?
76. The propeller in the previous problem slows from 475 rev/min to 187 rev/min in 4.00 s. What is its angular acceleration?
77. An automobile wheel with a 9.00 cm radius, as shown in **Figure 8-23**, rotates at 2.50 rad/s. How fast does a point 7.00 cm from the center travel?



■ Figure 8-23

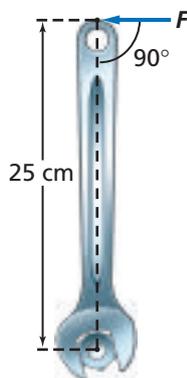
78. **Washing Machine** A washing machine's two spin cycles are 328 rev/min and 542 rev/min. The diameter of the drum is 0.43 m.
- What is the ratio of the centripetal accelerations for the fast and slow spin cycles? Recall that $a_c = \frac{v^2}{r}$ and $v = rw$
 - What is the ratio of the linear velocity of an object at the surface of the drum for the fast and slow spin cycles?
79. Find the maximum centripetal acceleration in terms of g for the washing machine in problem 78.

Chapter 8 Assessment

80. A laboratory ultracentrifuge is designed to produce a centripetal acceleration of $0.35 \times 10^6 g$ at a distance of 2.50 cm from the axis. What angular velocity in rev/min is required?

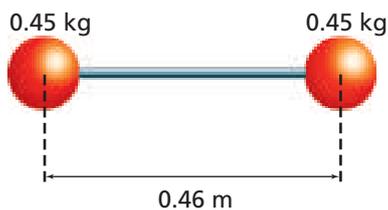
8.2 Rotational Dynamics

81. **Wrench** A bolt is to be tightened with a torque of $8.0 \text{ N}\cdot\text{m}$. If you have a wrench that is 0.35 m long, what is the least amount of force you must exert?
82. What is the torque on a bolt produced by a 15-N force exerted perpendicular to a wrench that is 25 cm long, as shown in **Figure 8-24**?



■ Figure 8-24

83. A toy consisting of two balls, each 0.45 kg, at the ends of a 0.46-m-long, thin, lightweight rod is shown in **Figure 8-25**. Find the moment of inertia of the toy. The moment of inertia is to be found about the center of the rod.

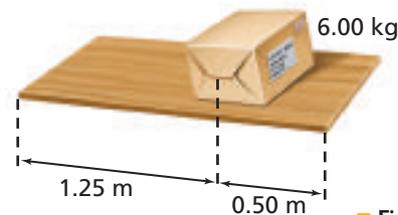


■ Figure 8-25

84. A bicycle wheel with a radius of 38 cm is given an angular acceleration of 2.67 rad/s^2 by applying a force of 0.35 N on the edge of the wheel. What is the wheel's moment of inertia?
85. **Toy Top** A toy top consists of a rod with a diameter of 8.0-mm and a disk of mass 0.0125 kg and a diameter of 3.5 cm. The moment of inertia of the rod can be neglected. The top is spun by wrapping a string around the rod and pulling it with a velocity that increases from zero to 3.0 m/s over 0.50 s.
- What is the resulting angular velocity of the top?
 - What force was exerted on the string?

8.3 Equilibrium

86. A 12.5-kg board, 4.00 m long, is being held up on one end by Ahmed. He calls for help, and Judi responds.
- What is the least force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force?
 - What is the greatest force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force?
87. Two people are holding up the ends of a 4.25-kg wooden board that is 1.75 m long. A 6.00-kg box sits on the board, 0.50 m from one end, as shown in **Figure 8-26**. What forces do the two people exert?

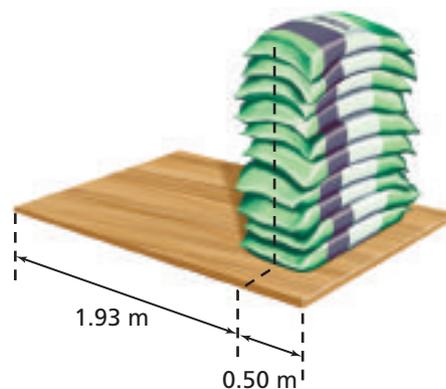


■ Figure 8-26

88. A car's specifications state that its weight distribution is 53 percent on the front tires and 47 percent on the rear tires. The wheel base is 2.46 m. Where is the car's center of mass?

Mixed Review

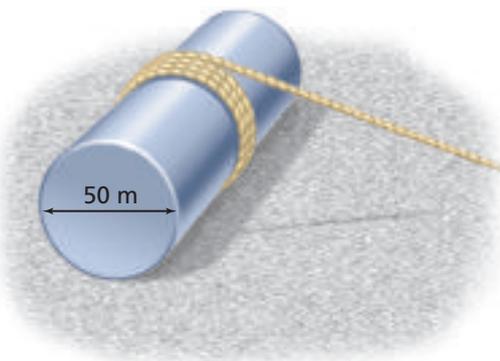
89. A wooden door of mass, m , and length, l , is held horizontally by Dan and Ajit. Dan suddenly drops his end.
- What is the angular acceleration of the door just after Dan lets go?
 - Is the acceleration constant? Explain.
90. **Topsoil** Ten bags of topsoil, each weighing 175 N, are placed on a 2.43-m-long sheet of wood. They are stacked 0.50 m from one end of the sheet of wood, as shown in **Figure 8-27**. Two people lift the sheet of wood, one at each end. Ignoring the weight of the wood, how much force must each person exert?



■ Figure 8-27



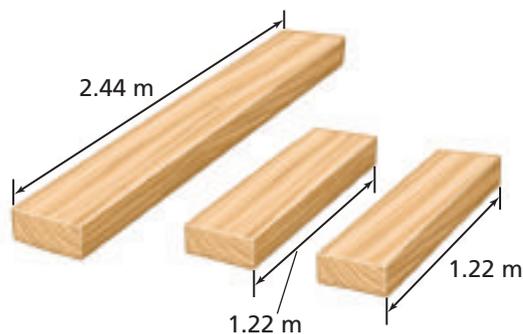
- 91. Basketball** A basketball is rolled down the court. A regulation basketball has a diameter of 24.1 cm, a mass of 0.60 kg, and a moment of inertia of $5.8 \times 10^{-3} \text{ kg}\cdot\text{m}^2$. The basketball's initial velocity is 2.5 m/s.
- What is its initial angular velocity?
 - The ball rolls a total of 12 m. How many revolutions does it make?
 - What is its total angular displacement?
- 92.** The basketball in the previous problem stops rolling after traveling 12 m.
- If its acceleration was constant, what was its angular acceleration?
 - What torque was acting on it as it was slowing down?
- 93.** A cylinder with a 50 m diameter, as shown in **Figure 8-28**, is at rest on a surface. A rope is wrapped around the cylinder and pulled. The cylinder rolls without slipping.
- After the rope has been pulled a distance of 2.50 m at a constant speed, how far has the center of mass of the cylinder moved?
 - If the rope was pulled a distance of 2.50 m in 1.25 s, how fast was the center of mass of the cylinder moving?
 - What is the angular velocity of the cylinder?



■ Figure 8-28

- 94. Hard Drive** A hard drive on a modern computer spins at 7200 rpm (revolutions per minute). If the drive is designed to start from rest and reach operating speed in 1.5 s, what is the angular acceleration of the disk?
- 95. Speedometers** Most speedometers in automobiles measure the angular velocity of the transmission and convert it to speed. How will increasing the diameter of the tires affect the reading of the speedometer?
- 96.** A box is dragged across the floor using a rope that is a distance h above the floor. The coefficient of friction is 0.35. The box is 0.50 m high and 0.25 m wide. Find the force that just tips the box.

- 97.** The second hand on a watch is 12 mm long. What is the velocity of its tip?
- 98. Lumber** You buy a 2.44-m-long piece of 10 cm \times 10 cm lumber. Your friend buys a piece of the same size and cuts it into two lengths, each 1.22 m long, as shown in **Figure 8-29**. You each carry your lumber on your shoulders.
- Which load is easier to lift? Why?
 - Both you and your friend apply a torque with your hands to keep the lumber from rotating. Which load is easier to keep from rotating? Why?



■ Figure 8-29

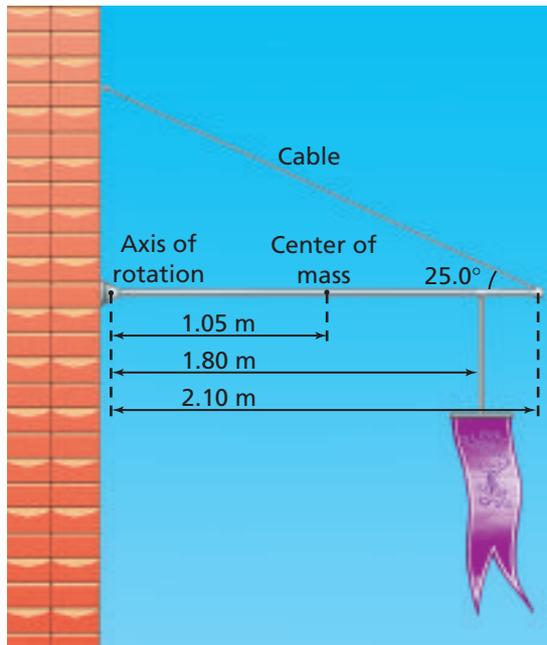
- 99. Surfboard** Harris and Paul carry a surfboard that is 2.43 m long and weighs 143 N. Paul lifts one end with a force of 57 N.
- What force must Harris exert?
 - What part of the board should Harris lift?
- 100.** A steel beam that is 6.50 m long weighs 325 N. It rests on two supports, 3.00 m apart, with equal amounts of the beam extending from each end. Suki, who weighs 575 N, stands on the beam in the center and then walks toward one end. How close to the end can she come before the beam begins to tip?

Thinking Critically

- 101. Apply Concepts** Consider a point on the edge of a rotating wheel.
- Under what conditions can the centripetal acceleration be zero?
 - Under what conditions can the tangential (linear) acceleration be zero?
 - Can the tangential acceleration be nonzero while the centripetal acceleration is zero? Explain.
 - Can the centripetal acceleration be nonzero while the tangential acceleration is zero? Explain.
- 102. Apply Concepts** When you apply the brakes in a car, the front end dips. Why?

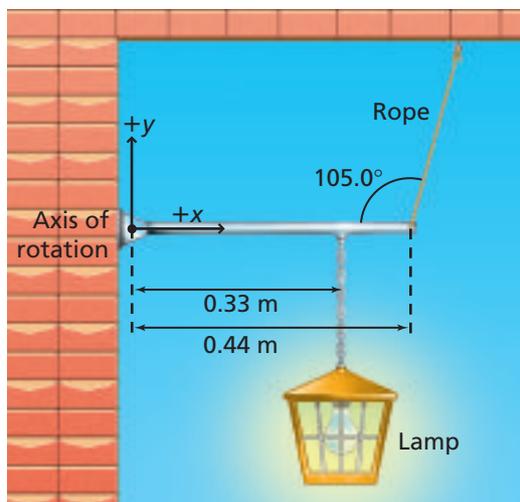
Chapter 8 Assessment

- 103. Analyze and Conclude** A banner is suspended from a horizontal, pivoted pole, as shown in **Figure 8-30**. The pole is 2.10 m long and weighs 175 N. The banner, which weighs 105 N, is suspended 1.80 m from the pivot point or axis of rotation. What is the tension in the cable supporting the pole?



■ **Figure 8-30**

- 104. Analyze and Conclude** A pivoted lamp pole is shown in **Figure 8-31**. The pole weighs 27 N, and the lamp weighs 64 N.
- What is the torque caused by each force?
 - Determine the tension in the rope supporting the lamp pole.



■ **Figure 8-31**

- 105. Analyze and Conclude** Gerald and Evelyn carry the following objects up a flight of stairs: a large mirror, a dresser, and a television. Evelyn is at the front end, and Gerald is at the bottom end. Assume that both Evelyn and Gerald exert only upward forces.
- Draw a free-body diagram showing Gerald and Evelyn exerting the same force on the mirror.
 - Draw a free-body diagram showing Gerald exerting more force on the bottom of the dresser.
 - Where would the center of mass of the television have to be so that Gerald carries all the weight?

Writing in Physics

- 106.** Astronomers know that if a satellite is too close to a planet, it will be torn apart by tidal forces. That is, the difference in the gravitational force on the part of the satellite nearest the planet and the part farthest from the planet is stronger than the forces holding the satellite together. Do research on the Roche limit and determine how close the Moon would have to orbit Earth to be at the Roche limit.
- 107.** Automobile engines are rated by the torque that they produce. Research and explain why torque is an important quantity to measure.

Cumulative Review

- 108.** Two blocks, one of mass 2.0 kg and the other of mass 3.0 kg, are tied together with a massless rope. This rope is strung over a massless, resistance-free pulley. The blocks are released from rest. Find the following. (**Chapter 4**)
- the tension in the rope
 - the acceleration of the blocks.
- 109.** Eric sits on a see-saw. At what angle, relative to the vertical, will the component of his weight parallel to the plane be equal to one-third the perpendicular component of his weight? (**Chapter 5**)
- 110.** The pilot of a plane wants to reach an airport 325 km due north in 2.75 hours. A wind is blowing from the west at 30.0 km/h. What heading and airspeed should be chosen to reach the destination on time? (**Chapter 6**)
- 111.** A 60.0-kg speed skater with a velocity of 18.0 m/s comes into a curve of 20.0-m radius. How much friction must be exerted between the skates and ice to negotiate the curve? (**Chapter 6**)

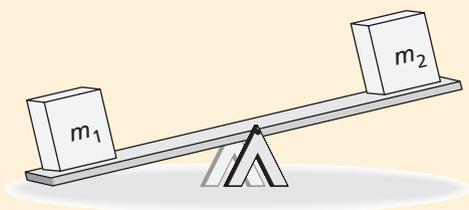


Standardized Test Practice

Multiple Choice

1. The illustration below shows two boxes on opposite ends of a board that is 3.0 m long. The board is supported in the middle by a fulcrum. The box on the left has a mass, m_1 , of 25 kg, and the box on the right has a mass, m_2 , of 15 kg. How far should the fulcrum be positioned from the left side of the board in order to balance the masses horizontally?

- (A) 0.38 m (C) 1.1 m
(B) 0.60 m (D) 1.9 m



2. A force of 60 N is exerted on one end of a 1.0-m-long lever. The other end of the lever is attached to a rotating rod that is perpendicular to the lever. By pushing down on the end of the lever, you can rotate the rod. If the force on the lever is exerted at an angle of 30° , what torque is exerted on the lever? ($\sin 30^\circ = 0.5$; $\cos 30^\circ = 0.87$; $\tan 30^\circ = 0.58$)

- (A) 30 N (C) 60 N
(B) 52 N (D) 69 N

3. A child attempts to use a wrench to remove a nut on a bicycle. Removing the nut requires a torque of $10 \text{ N}\cdot\text{m}$. The maximum force the child is capable of exerting at a 90° angle is 50 N. What is the length of the wrench the child must use to remove the nut?

- (A) 0.1 m (C) 0.2 m
(B) 0.15 m (D) 0.25 m

4. A car moves a distance of 420 m. Each tire on the car has a diameter of 42 cm. Which of the following shows how many revolutions each tire makes as they move that distance?

- (A) $\frac{5.0 \times 10^1}{\pi}$ rev (C) $\frac{1.5 \times 10^2}{\pi}$ rev
(B) $\frac{1.0 \times 10^2}{\pi}$ rev (D) $\frac{1.0 \times 10^3}{\pi}$ rev

5. A thin hoop with a mass of 5.0 kg rotates about a perpendicular axis through its center. A force of 25 N is exerted tangentially to the hoop. If the hoop's radius is 2.0 m, what is its angular acceleration?

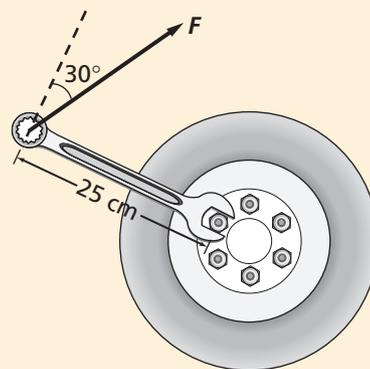
- (A) 1.3 rad/s (C) 5.0 rad/s
(B) 2.5 rad/s (D) 6.3 rad/s

6. Two of the tires on a farmer's tractor have diameters of 1.5 m. If the farmer drives the tractor at a linear velocity of 3.0 m/s, what is the angular velocity of each tire?

- (A) 2.0 rad/s (C) 4.0 rad/s
(B) 2.3 rad/s (D) 4.5 rad/s

Extended Answer

7. You use a 25-cm long wrench to remove the lug nuts on a car wheel, as shown in the illustration below. If you pull up on the end of the wrench with a force of $2.0 \times 10^2 \text{ N}$ at an angle of 30° , what is the torque on the wrench? ($\sin 30^\circ = 0.5$, $\cos 30^\circ = 0.87$)



✓ Test-Taking TIP

When Eliminating, Cross It Out

Consider each answer choice individually and cross out the ones you have eliminated. If you cannot write in the test booklet, use the scratch paper to list and cross off the answer choices. You will save time and stop yourself from choosing an answer you have mentally eliminated.

